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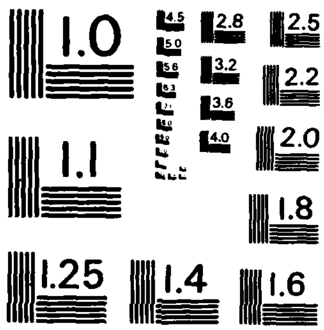

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COORDINATED SCIENCE LABORATORY
College of Engineering

DIVERSITY COMBINING FOR FREQUENCY-HOP SPREAD-SPECTRUM COMMUNICATIONS WITH PARTIAL-BAND INTERFERENCE AND FADING

Catherine M. Keller

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

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This report presents results on the evaluation of several diversity combining techniques that are suggested for frequency-hop (FH) communications with partial-band interference and fading. The analysis covers systems with M-ary orthogonal signaling and noncoherent demodulation. The partial-band interference is modeled as a Guassian process, although some of the results also apply to general (non-Gaussian) partial-band interference. The performance measures we use to evaluate the diversity combining techniques are the narrowband interference rejection capability and the signal to noise ratio requirement over the entire range of interference duty factors. We evaluate the exact probability of error for each of the diversity combining techniques studied.

The performance of the optimum combining technique for receivers with perfect side information is established. It is shown that for receivers with perfect side information, the system performance does not change significantly with the choice of the diversity combining technique. However, the same schemes that work well in receivers with perfect side information perform poorly in receivers without side information. The goal of this work is to

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Clipped linear combining is proposed as a diversity combining technique for receivers without side information. The numerical results demonstrate that clipped linear combining can perform nearly as well as the optimum combining technique in terms of both narrowband interference rejection and signal to noise ratio requirement. However, knowledge of the signal output voltage is required to set the clipping level. We analyze two alternative diversity combining techniques that do not have this requirement. These diversity combining schemes use ratio statistics in a ratio threshold test to determine the quality of each diversity reception. It is shown that the ratio threshold test with diversity combining provides good narrowband interference rejection, but at the expense of an increased signal to noise ratio requirement near full-band interference. Although the ratio threshold test with diversity combining does not achieve the optimum performance, it is an effective, as well as practical, scheme for use in FH communication systems with partial-band interference and fading.

DIVERSITY COMBINING FOR FREQUENCY-HOP SPREAD-SPECTRUM COMMUNICATIONS WITH PARTIAL-BAND INTERFERENCE AND FADING

BY

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B.S., Carnegie-Mellon University, 1980
M.S., University of Illinois, 1983

THESIS

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DIVERSITY COMBINING FOR FREQUENCY-HOP SPREAD-SPECTRUM COMMUNICATIONS WITH PARTIAL-BAND INTERFERENCE AND FADING

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Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign, 1985

ABSTRACT

This thesis presents results on the evaluation of several diversity combining techniques that are suggested for frequency-hop (FH) communications with partial-band interference and fading. The analysis covers systems with M-ary orthogonal signaling and noncoherent demodulation. The partial-band interference is modeled as a Gaussian process, although some of the results also apply to general (non-Gaussian) partial-band interference. The performance measures we use to evaluate the diversity combining techniques are the narrowband interference rejection capability and the signal to noise ratio requirement over the entire range of interference duty factors. We evaluate the exact probability of error for each of the diversity combining techniques studied.

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For Shaun

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CHAPTER 1

INTRODUCTION

Diversity transmission is often employed to provide reliable communication in the presence of fading or partial-band interference. We consider a system in which the diversity receptions are first demodulated by a noncoherent matched filter followed by an envelope detector; this is equivalent to the square root of the sum of the squares of the outputs of an inphase-quadrature (I-Q) square-law demodulator. The envelope detector outputs corresponding to the diversity receptions of a given data symbol are then combined in some way to form the decision statistics for the receiver. Equivalent block diagrams for the demodulator and diversity combiner are shown in Figures 1.1 and 1.2. Except for I-Q magnitude-law combining, all of the diversity combining techniques considered in this thesis fit these models.

The performance of the diversity transmission system depends on the way in which the diversity receptions are combined. The purpose of this research is to investigate various diversity combining techniques for applications to noncoherent frequency-hop (FH) communication systems with partial-band interference and fading.

Some of the methods for combining the diversity receptions are only useful when side information is available at the receiver. By *side information*, we mean information concerning the presence or absence of interference on a given diversity reception. Other studies have shown that when side information is available, coding and diversity may be used to virtually eliminate the effects of narrowband interference. Many codes have been studied [1]-[5], but Reed-Solomon coding is particularly attractive for this application.

Square-law combining has been studied extensively for use in noncoherent systems. This is in part because it is the optimum noncoherent combining technique for Rayleigh fading [6]-[7], and in part because its analysis is easier than most other combining methods. However, square-law combining is not optimum for other types of fading or for channels with partial-

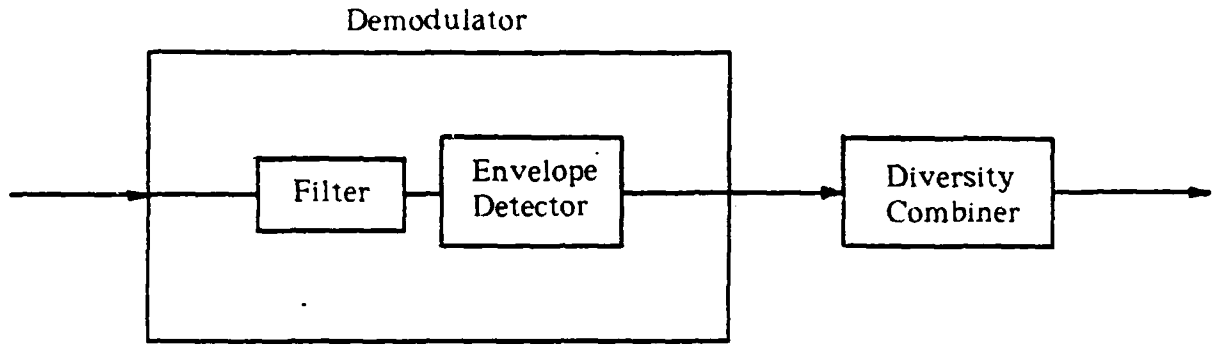

Figure 1.1. Block diagram of one branch of a receiver employing diversity combining

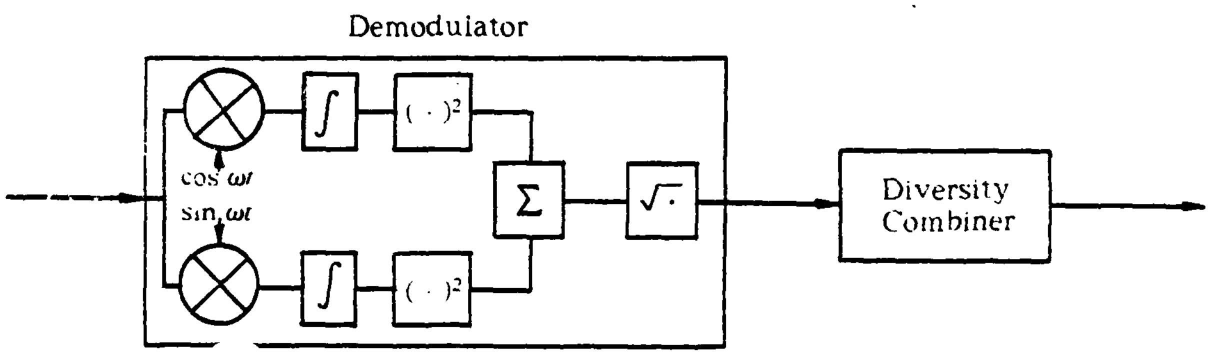

Figure 1.2. Equivalent block diagram of one branch of a receiver employing diversity combining band interference. In Chapter 2, we study the optimum noncoherent combining technique for Gaussian partial-band interference, as well as four suboptimal combining schemes including square-law combining. Square-law combining, linear combining, square-root combining, I-Q magnitude-law combining, and optimum combining are compared for receivers with side information.

Side information is not always available at the receiver. The requirement to extract side information increases the receiver complexity, and there is always concern about the reliability of the side information. Some diversity techniques that work well with side information perform poorly when there is no side information. Because of these considerations, diversity combining techniques that do not require side information from external sources are particularly attractive. In one such diversity combining scheme, called *clipped linear combining*, the envelope detector outputs of each diversity reception are clipped before they are combined. The role of the clipper is to constrain the effects of strong narrowband interference. Clipped linear combining is analyzed in Chapter 3.

Although clipped linear combining is very effective against partial-band interference, there is a practical disadvantage to this diversity combining scheme. The clipping level depends on the signal output voltage (i. e., the envelope detector output voltage due to signal only). This signal output voltage may be difficult to measure in practice.

It is desirable to employ diversity combining techniques that do not depend on the received signal power. One such diversity combining technique uses Viterbi's *ratio threshold test* [8]-[10]. In the ratio threshold test, the ratio statistic for a given diversity reception is the ratio of the largest envelope detector output to the second largest envelope detector output. We discuss several diversity combining techniques that use the ratio threshold test.

In the system that uses the ratio threshold test with *linear combining*, a diversity reception is rejected if its ratio statistic is less than a prescribed threshold. If the ratio is greater than the threshold, the diversity reception is accepted. If at least one diversity reception is accepted, then only the accepted diversity receptions are combined. If all diversity receptions are rejected, then all of them are combined and a hard decision is made. We analyze the ratio threshold test with linear combining for systems with binary orthogonal signaling.

An alternative scheme is to make a hard decision on each accepted diversity reception. This is followed by *majority logic decoding* of all accepted diversity receptions [10]. If a tie occurs among the accepted diversity receptions, or if all diversity receptions are rejected, a random guess is made.

Improvement is possible for the ratio threshold test with either linear combining or majority logic decoding by employing other strategies if all the diversity receptions are rejected or if a tie occurs. In these situations, the ratio statistics for the diversity receptions may be used again. For both linear combining and majority logic decoding, we examine the strategy of basing the decision on the diversity reception with the largest ratio for the situation in which all diversity receptions are rejected.

The ratio threshold test is also considered for use in systems with M-ary orthogonal signaling. A hard decision is made on each diversity reception. The ratio threshold test is employed to determine which diversity receptions to include in the decision process. Then, the symbol with the most decisions in its favor, out of the accepted diversity receptions, is chosen. These diversity combining techniques based on the ratio statistic are examined in Chapter 4, where they are compared to clipped linear combining and optimum combining.

For partial-band interference, the interference *duty factor* is the fraction of the frequency band of the FH system that is occupied by the interference. Similarly, the interference duty factor represents the fraction of the time that partial-time interference is present in a system which uses time diversity. That is, although we give results for FH systems with partial-band interference, the results also apply to systems employing time diversity, achieved by interleaving, with partial-time interference.

Much of the work done on the analysis of FH systems with partial-band interference has dealt with the the worst-case system performance [3]-[5], [8]-[14]; intentional partial-band jamming is the main concern in these studies. The worst-case duty factor for the interference and the corresponding system performance depend on the diversity level, the signal to noise ratio, the code used, and the specific diversity combining technique employed by the system. The analysis of the worst-case situation is rather difficult. In an earlier study [4], the worst-case interference duty factor was approximated by the interference duty factor that maximizes the Chernoff bound for the probability of error. This approximation of the worst-case interference has been used in other works, such as [5] and [12]. However, it has been shown that the Chernoff bound is not tight [2], [13], and that the exact value for the worst-case partial-band interference duty factor is not the same as the Chernoff bound value [14].

In this research, we examine how well the diversity combining system performs as a function of the duty factor of the partial-band interference. This type of analysis is motivated in part by the fact that partial-band interference is not always due to hostile jamming. For example, narrowband transmitters that operate in the same frequency band as the FH system are a source of partial-band interference. Multiple access interference is another source of partial-band interference [1]. We are interested in combining techniques that mitigate the effects of unintentional jamming and that force a hostile jammer to adopt expensive strategies in order to be effective.

There are two primary system performance measures that we use throughout this thesis. For each value of the partial-band interference duty factor, we compute the signal to noise ratio required to achieve a given error probability. The maximum signal to noise ratio required over the range of duty factors is one system performance measure of interest. It is desirable to minimize this maximum signal to noise ratio; at the same time, it is desirable for the maximum to occur at as large an interference duty factor as possible. We would like the worst-case duty factor to be unity without significantly increasing the maximum required signal to noise ratio

[1], [2]. In practice, full-band jamming of a wideband spread-spectrum communications system is more expensive than narrowband jamming.

We are also interested in the system's narrowband interference rejection capability. This is measured in terms of the largest number, such that a given error probability can be achieved in the presence of interference for any duty factor less than that number. A system has good narrowband interference rejection if this number is large.

In addition to partial-band interference, the communications channel may exhibit fading. An analysis of noncoherent communications in the presence of nonselective Rician and Rayleigh fading and partial-band interference was given in [1]. For the system with diversity, side information was assumed to be available and square-law combining was used. In Rayleigh fading, this is the optimum scheme to use. Chapter 5 is devoted to an extension of the results of [1]. We examine the performance of the diversity combining schemes proposed in Chapter 4 for channels with partial-band interference and nonselective fading. The purpose of this study is to demonstrate how well these diversity combining schemes perform in a fading environment.

CHAPTER 2

DIVERSITY COMBINING FOR RECEIVERS WITH SIDE INFORMATION

In this chapter, we discuss diversity combining techniques for systems in which perfect side information is available at the receiver. I-Q square-law combining, which has been studied for this application in [1]-[2], [11]-[12], is compared with linear combining and square-root combining. We also introduce the suboptimum I-Q magnitude-law receiver and describe I-Q magnitude-law combining. One motivation for these alternative diversity combining schemes is their superiority in the presence of a one-dimensional *tone jammer*. Consider a system which employs M-ary frequency-shift keying (FSK). A one-dimensional tone jammer affects only one of the M-ary tones on a given diversity reception. The jammer is present on a given diversity reception with probability ρ. The jammer power is fixed, so that when the jammer is present, the power applied is the average jammer power divided by ρ. In square-law combining, the decision statistics are the sums of squares. In linear combining, the decision statistics are the sums of linear terms. Given that the jammer power applied to a diversity reception overwhelms the signal power, especially if ρ is small, a jammed diversity reception will be given more emphasis in a sum of squares than in a sum of linear terms. Thus, it seems, intuitively at least, that linear combining may be better than square-law combining in a jamming environment.

Another motivation for employing these alternative diversity combining schemes is that for certain applications linear combining and I-Q magnitude-law combining are easier to implement than square-law combining. Unfortunately, they are more difficult to analyze than square-law combining.

None of the combining schemes discussed so far is the optimum combining scheme for Gaussian partial-band interference. In this chapter, we include a description of the combining scheme that is optimum for a system with perfect side information. The performance of the suboptimum combining schemes are compared with the performance of the optimum combining scheme.

2.1 System Model

We consider a noncoherent system with M-ary orthogonal signaling and diversity L. The interference is additive Gaussian noise and is present on a given symbol with probability ρ. With probability $1-\rho$, the symbol is received with no interference. This model is based on the assumption that the quiescent noise level due to thermal noise or other wideband noise sources is negligible. The partial-band interference is the primary source of noise. In a frequency-hop system, ρ represents the fractional bandwidth occupied by the partial-band interference. The noise power spectral density is $\frac{1}{2}\rho^{-1}N_I$ across that fraction of the band. Thus, the average power spectral density is $\frac{1}{2}N_I$. If $\rho=1$, the channel is the additive white Gaussian noise (AWGN) channel with two-sided power spectral density $\frac{1}{2}N_I$.

When diversity and side information are employed, the receiver can ignore the diversity receptions that have interference, and it can extract the data from the noise-free diversity receptions. If all the diversity receptions of a symbol are noisy, they are combined and a decision is made on which M-ary symbol was sent. For this model, symbol errors are possible only when all diversity receptions of the given symbol have interference. The interference is present on all of the diversity receptions with probability ρ^L.

Given that all diversity receptions have interference present, the diversity combiner in the receiver takes each demodulator output, processes it in some way, and then adds the processed diversity receptions. For example, in square-law combining and in linear combining, the demodulator consists of a filter followed by an envelope detector. In square-law combining, the demodulated diversity receptions are squared before they are added. In linear combining, the demodulated diversity receptions are added directly. Block diagrams for a receiver with square-law combining and for a receiver with linear combiner are shown in Figures 2.1 and 2.2.

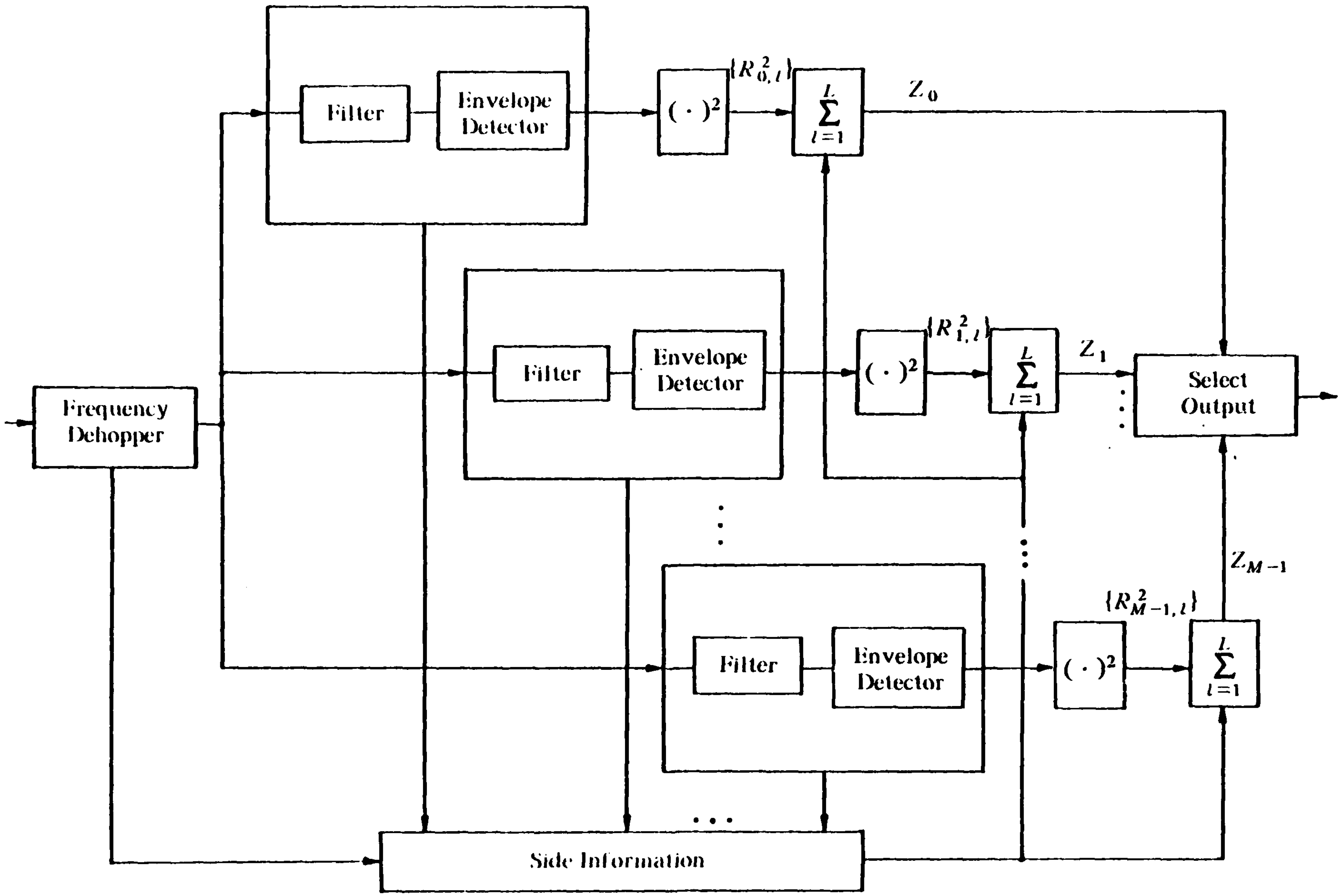

Figure 2.1. Block diagram of a receiver employing square-law combining

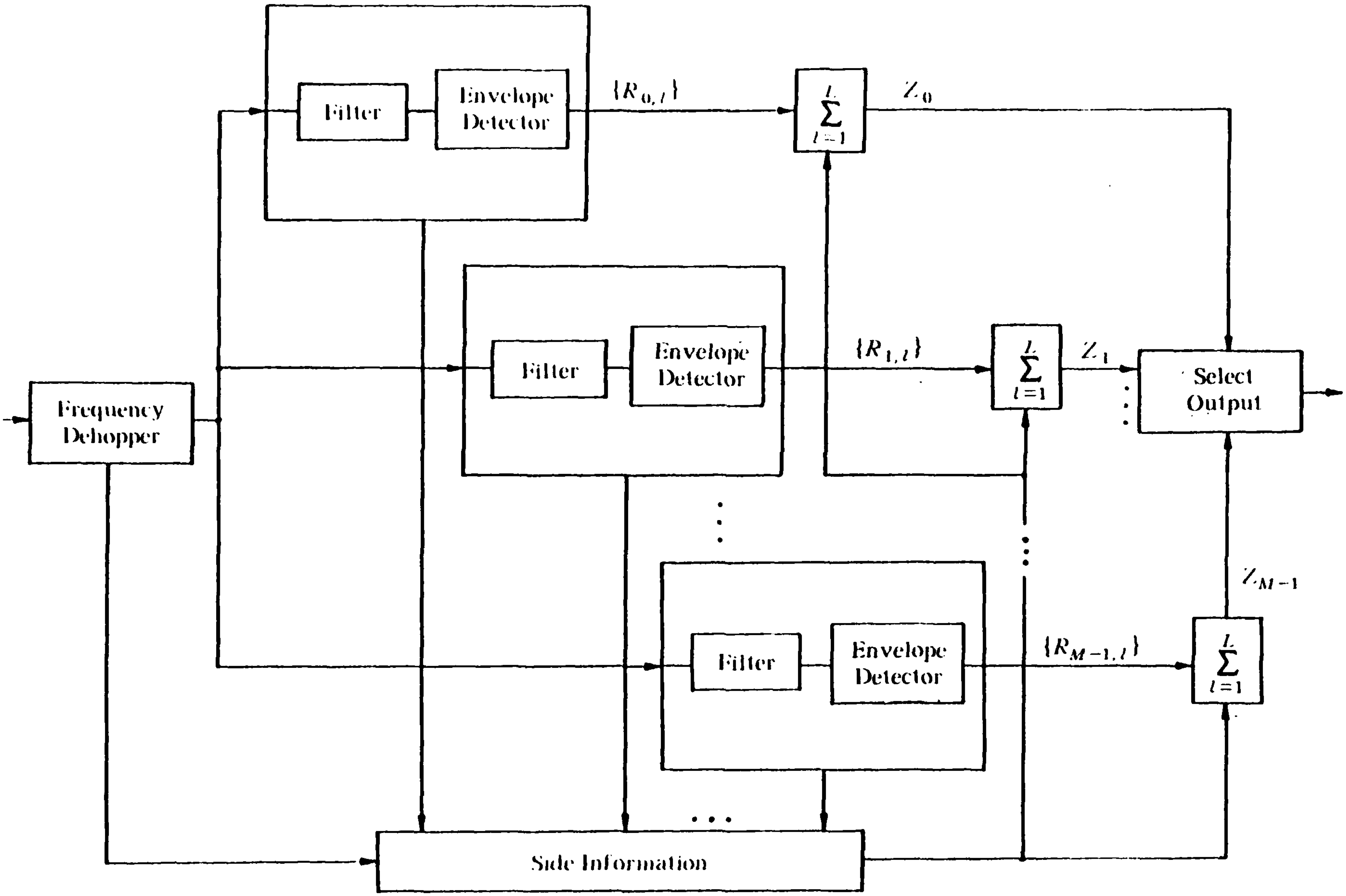

Figure 2.2. Block diagram of a receiver employing linear combining

Given that the symbol 0 is sent, the symbol error probability for the diversity system with perfect side information can be written as [2]

$$p_s = \rho^L \left[1 - \int_0^\infty f_0^{(L)}(x) \left[\int_0^x f_k^{(L)}(y)dy \right]^{M-1} dx \right]. \tag{2.1}$$

For each k, the densities $f_k^{(L)}(x)$ are the density functions of the outputs of the diversity combiner. These densities are the $(L-1)$-fold self-convolution of $f_k^{(1)}(x)$, which are the density functions for the processed diversity receptions before the combiner takes their sum. (An r-fold self-convolution of f is f if $r=0$, it is $f*f$ if $r=1$, it is $f*f*f$ if $r=2$, etc.) These density functions depend on the combining technique employed in the system.

For a given symbol error probability p_s, the parameter ρ^* is such that for all ρ less than ρ^*, p_s can be achieved regardless of the type of interference [1]-[2]. The parameter ρ^* does not depend on the statistical distribution or the power level of the interference. It is desirable to have ρ^* as close to unity as possible, indicating that the system can eliminate detrimental effects of narrowband interference. For a system with side information but no coding, the value of ρ^* for diversity L and symbol error probability p_s is $p_s^{1/L}$. All the diversity combining schemes studied in this chapter have $\rho^* = p_s^{1/L}$.

2.2 A Description of Several Diversity Combining Techniques

We compare square-law combining, linear combining, square-root combining, and I-Q magnitude-law combining employed in diversity systems in the presence of Gaussian partial-band interference with duty factor ρ. If interference is present on all L diversity receptions of a transmitted symbol, then all L diversity receptions are combined. A decision is made based on the statistic that is the combination of the diversity receptions.

In a system employing square-law combining, the squares of the outputs of the envelope detectors are added [2]. This is equivalent to adding the outputs of an I-Q square-law detector. Figure 2.1 shows a block diagram of a receiver using square-law combining. Given that the symbol 0 is sent, the decision statistics are

$$Z_0 = \sum_{l=1}^{L} (X_{0,l} + \nu)^2 + Y_{0,l}^2 \tag{2.2a}$$

$$Z_k = \sum_{l=1}^{L} X_{k,l}^2 + Y_{k,l}^2, \quad 1 \leqslant k \leqslant M-1 \tag{2.2b}$$

where

$$\begin{aligned} \nu &= \sqrt{2(E_s/N_I)\rho/L} \\ &= \sqrt{2 \log_2 M \ (E_b/N_I)\rho/L}\,. \end{aligned} \tag{2.3}$$

The quantity E_s is the symbol energy, E_b is the bit energy, and E_s/L and E_b/L are the symbol energy and bit energy per diversity transmission. $\{X_{k,l}, Y_{k,l} : 0 \leqslant k \leqslant M-1, 1 \leqslant l \leqslant L\}$ are mutually independent zero-mean, unit-variance Gaussian random variables. The densities for Z_k, $0 \leqslant k \leqslant M-1$ are well-known and are given by [15]

$$f_0^{(L)}(x) = \frac{1}{2}\left(\frac{x}{\nu^2 L}\right)^{\frac{L-1}{2}} \exp\left(\frac{-x+\nu^2 L}{2}\right) I_{L-1}(\sqrt{x\nu^2 L}\,), \quad x > 0, \tag{2.4a}$$

and

$$f_k^{(L)}(y) = \frac{y^{L-1}}{2^L (L-1)!} \exp\left(-\frac{y}{2}\right), \quad y \geqslant 0, \ 1 \leqslant k \leqslant M-1, \tag{2.4b}$$

where $I_\nu(\cdot)$ is the ν-th order modified Bessel function.

For *linear combining*, the decision statistics are the sums of the outputs of the envelope detectors. A block diagram of a receiver with linear combining is shown in Figure 2.2. Given the symbol 0 is sent, the decision statistics are

$$Z_0 = \sum_{l=1}^{L} \sqrt{(X_{0,l} + \nu)^2 + Y_{0,l}^2} \tag{2.5a}$$

$$Z_k = \sum_{l=1}^{L} \sqrt{X_{k,l}^2 + Y_{k,l}^2}\,, \quad 1 \leqslant k \leqslant M-1. \tag{2.5b}$$

Thus, Z_k is the sum of L Rician distributed random variables for $k=0$, and Z_k is the sum of L Rayleigh distributed random variables for $k>0$. For $L>2$, closed form analytical expressions for the densities of Z_k are not known; they are found by using numerical techniques.

$$+\exp[-(x+v(\sin\theta-\cos\theta))^2/4]\,[Q(\frac{-v(\cos\theta+\sin\theta)-x}{\sqrt{2}})-Q(\frac{-v(\cos\theta+\sin\theta)+x}{\sqrt{2}})] \qquad (2.8a)$$

$$+\exp[-(x+v(\sin\theta+\cos\theta))^2/4]\,[Q(\frac{-v(\cos\theta-\sin\theta)-x}{\sqrt{2}})-Q(\frac{-v(\cos\theta-\sin\theta)+x}{\sqrt{2}})]\}d\theta,\ x\geq 0,$$

and

$$f_k^{(1)}(y)=\frac{4}{\sqrt{\pi}}e^{-y^2/4}\ [\frac{1}{2}-Q(\frac{y}{\sqrt{2}})],\ y\geq 0,\ 1\leq k\leq M-1, \qquad (2.8b)$$

where $Q(\cdot)$ is the complementary cumulative Gaussian distribution function. The densities for the statistics in (2.7) are the $(L-1)$-fold self-convolutions of $f_k^{(1)}(x)$ for each k, and are computed with numerical techniques.

Consider the probability of error for the I-Q magnitude-law *receiver* with diversity level 1 in AWGN. This performance is not widely known, so we briefly discuss it here. That is, we discuss the performance of the system with no diversity and with $\rho=1$. We compute the exact average probability of error given the densities in (2.8). Also, we compute the worst-case performance for the I-Q magnitude-law receiver which is found by substituting $\theta=\frac{\pi}{2}i$ for $i=0, 1, 2,$ or 3 in the densities of (2.8). That is, the minimum expected value of Z_0 in (2.7a) occurs at any of these values of θ. In Figures 2.4, 2.5, and 2.6, we compare the probability of error of the I-Q square-law receiver to that of the I-Q magnitude-law receiver in AWGN ($\rho=1$) for $L=1$, and $M=2$, 16, and 64. Notice that over the range of signal to noise ratios shown, the I-Q magnitude-law receiver performance is not much more than 1 dB worse than the I-Q square-law receiver performance.

2.3 Optimum Diversity Combining

Consider, once again, diversity levels greater than or equal to 1. The decision statistics for the optimum diversity combining technique of envelope detector outputs in AWGN are known because of their application in radar detection problems, and they are easily derivable [16]-[19]. Given that the symbol 0 is sent, the decision statistics are

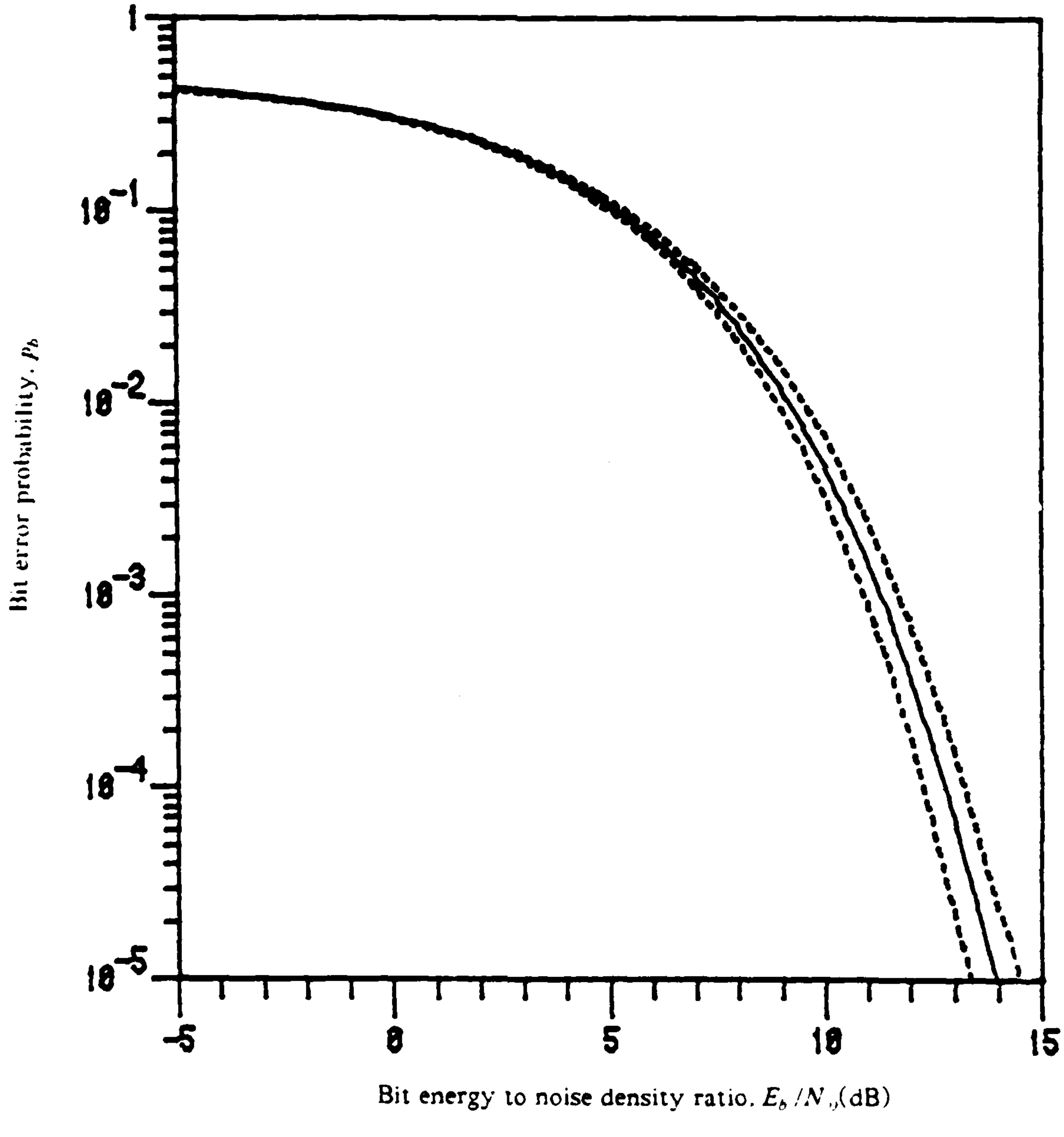

Figure 2.4. Bit error probability versus signal to noise ratio for the I-Q magnitude-law receiver with $M=2$ in AWGN ($\rho=1$), bounded below by the performance of the I-Q square-law receiver and bounded above by the worst case I-Q magnitude-law receiver performance

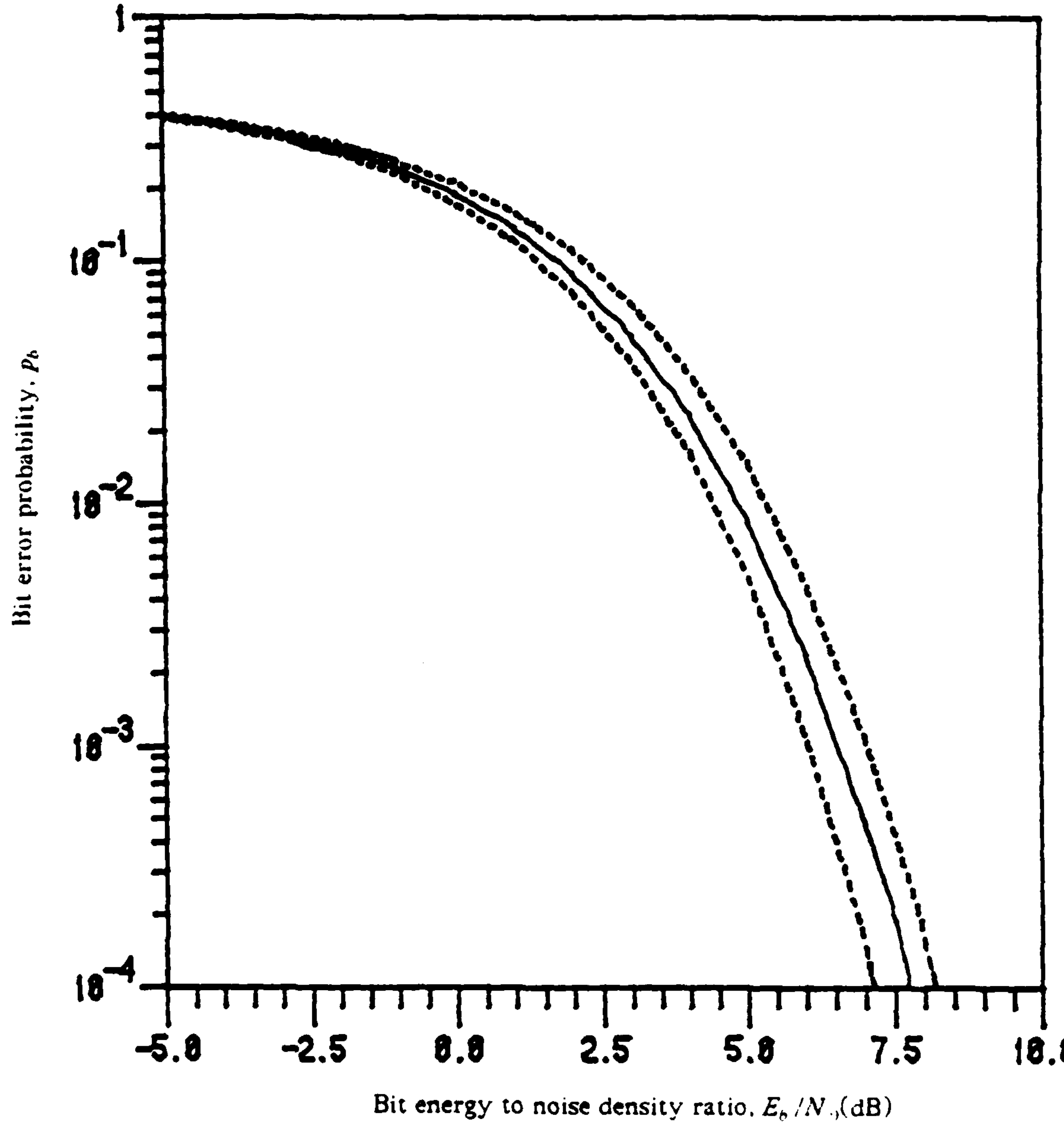

Figure 2.5. Bit error probability versus signal to noise ratio for the I-Q magnitude-law receiver with $M=16$ in AWGN ($\rho=1$), bounded below by the performance of the I-Q square-law receiver and bounded above by the worst case I-Q magnitude-law receiver performance

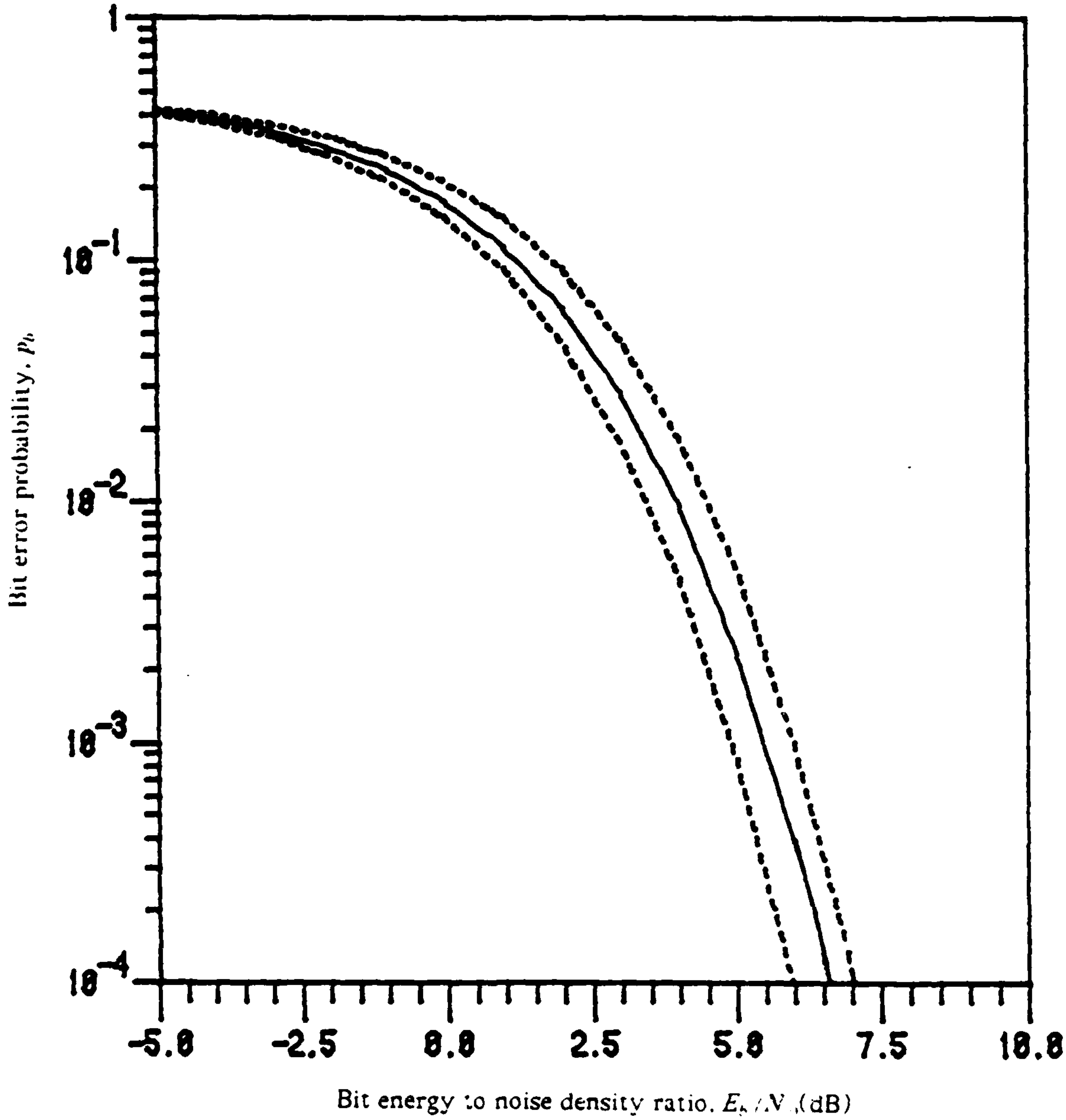

Figure 2.6. Bit error probability versus signal to noise ratio for the I-Q magnitude-law receiver with $M=64$ in AWGN ($\rho=1$), bounded below by the performance of the I-Q square-law receiver and bounded above by the worst case I-Q magnitude-law receiver performance

$$\zeta_0 = \prod_{l=1}^{L} I_0(\nu\sqrt{(X_{0,l}+\nu)^2+Y_{0,l}^2}) \tag{2.9a}$$

$$\zeta_k = \prod_{l=1}^{L} I_0(\nu\sqrt{X_{k,l}^2+Y_{k,l}^2}) \ , \ \ 1 \leqslant k \leqslant M-1. \tag{2.9b}$$

Now consider partial-band interference with duty factors other than $\rho=1$. The decision statistics in (2.9) are also the optimum decision statistics for a system with perfect side information in the presence of Gaussian partial-band interference of any duty factor. This is true, because, for systems with perfect side information, interference must be present on all diversity receptions in order for the diversity receptions to be combined. The situation in which interference is present on all diversity receptions corresponding to a given bit is analogous to the situation in which AWGN with double-sided power spectral density $\frac{1}{2}\rho^{-1}N_I$ is present on that bit.

By taking the natural logarithm of the decision statistics in (2.9), we obtain an alternate form of the decision statistics, namely

$$Z_0 = \sum_{l=1}^{L} \ln(I_0(\nu R_{0,l})) \tag{2.10a}$$

$$Z_k = \sum_{l=1}^{L} \ln(I_0(\nu R_{k,l})) \ , \ \ 1 \leqslant k \leqslant M-1. \tag{2.10b}$$

where $\{R_{0,l} : 1 \leqslant l \leqslant L\}$ are the outputs of the envelope detectors matched to the signal (Rician distributed random variables), and $\{R_{k,l} : 1 \leqslant k \leqslant M-1, 1 \leqslant l \leqslant L\}$ are the outputs of the envelope detectors with noise only (Rayleigh distributed random variables). Notice that these optimum decision statistics depend explicitly on the signal to noise ratio through the parameter ν, given in (2.3). This implies that the knowledge of the received signal to noise ratio is needed in order to apply the optimum combining technique.

For small values for its argument, the natural logarithm of the 0-th order modified Bessel function may be written as

$$\ln(I_0(x)) = \frac{1}{4}x^2 - \frac{1}{64}x^4 + O(x^6), \tag{2.11}$$

and for large values for its argument, it may be written as

$$\ln(I_0(x)) = x - \frac{1}{2}\ln(2\pi x) + (8x)^{-1} + O(x^{-2}). \tag{2.12}$$

Thus, when the outputs of the envelope detectors are small or the signal to noise ratio is small, the optimum combining technique approximates square-law combining. When the envelope detector outputs are large or the signal to noise ratio is large, the optimum combining technique approximates linear combining.

The symbol error probability for the optimum diversity combining technique is expressed in (2.1). For optimum combining, the densities $f_k^{(L)}(x)$ are the $(L-1)$-fold self-convolution of $f_k^{(1)}(x)$, which are given by

$$f_0^{(1)}(x) = \frac{I_0^{-1}(e^x)}{v^2 I_1(I_0^{-1}(e^x))}\exp\{-\frac{[I_0^{-1}(e^x)]^2 - 4xv^2 + v^4}{2v^2}\},\ x \geq 0, \tag{2.13a}$$

and

$$f_0^{(1)}(x) = \frac{I_0^{-1}(e^x)}{v^2 I_1(I_0^{-1}(e^x))}\exp\{-\frac{[I_0^{-1}(e^x)]^2 - 2xv^2}{2v^2}\},\ x \geq 0,\ 1 \leq k \leq M-1. \tag{2.13b}$$

2.4 Numerical Results

We evaluate (2.1) by numerical methods for each of the diversity combining schemes discussed. The 0-th order and 1-st order modified Bessel functions are calculated by using the polynomial approximations given in [20]. The inverse of the 0-th order modified Bessel function, needed in (2.13), is found by using iteration. The accuracy of this method is checked by comparing computer generated results with the tables in [20]. Other orders of modified Bessel functions, required for (2.4), are computed by using the integral definition for modified Bessel functions. The asymptotic approximations and polynomial approximations to $Q(\cdot)$ given in [20] are used to calculate the densities in (2.8). To compute the convolutions of the density functions and to compute the probability integrals, we use an array processor, which significantly speeds up the computations. See the Appendix for more information on the computational tools we use and how the data calculated for this thesis are verified.

In Table 2.1, we give values for the bit energy to noise ratio necessary to achieve a symbol error probability p_s of 0.1 for the example with $L=3$ and $M=32$ for square-law combining, linear combining, square-root combining, I-Q magnitude-law combining, and optimum combining. We use a symbol error probability of 0.1 in this and many other examples, because, although we are discussing system performance for uncoded systems, we assume that in practical applications some form of coding will be used. For example, if a (32, 10) extended Reed-Solomon code is used, $p_s=0.1$ corresponds to a bit error probability of $6.48 \cdot 10^{-6}$. For a system employing a (32, 16) Reed-Solomon code, the corresponding bit error probability is $4.95 \cdot 10^{-4}$.

As illustrated in Table 2.1, the performance is about the same for each of the combining schemes analyzed in this chapter. By examining the results for the four diversity combining techniques other than optimum combining, we see that linear combining is the best of the four techniques over a large range of values of ρ. Square-law combining is the best of these four over the rest of the range. From (2.11) and (2.12), we gain an intuitive explanation for the reason why there is a crossover in the performance of square-law combining and linear combining in Table 2.1. Linear combining performs better at larger values of ρ, where the signal to noise ratio of the operating point is large. (Since the system performance is based on symbol detection, the symbol energy to noise density ratio should be considered as the argument in the asymptotic analysis. The symbol energy to noise density ratio is found by adding $\log_2 32 \approx 7$dB to the values in Table 2.1.) Square-law combining does better for smaller values of ρ where the required signal to noise ratio is smaller. Indeed, by now examining the optimum combining results, we see that linear combining and optimum combining have nearly the same performance near $\rho=1$. Square-law combining and optimum combining have nearly the same performance for ρ close to ρ^*.

Consider the two performance measurements discussed in Chapter 1. There is no improvement in ρ^* for the optimum combining technique, because ρ^* is the same for all of the diversity combining techniques with perfect side information discussed in this chapter. Also,

TABLE 2.1

Bit energy to noise ratio (in dB) required to achieve a symbol error probability of $p_s = 0.1$ for $M = 32$ and diversity level $L = 3$.

ρ	Square-Law	Linear	Square-Root	Magnitude-Law	Optimum
0.475	-5.87	-5.79	-5.63	-5.65	-5.87
0.500	-0.62	-0.64	-0.51	-0.46	-0.66
0.600	2.50	2.42	2.52	2.61	2.41
0.700	3.19	3.09	3.18	3.29	3.08
0.800	3.41	3.31	3.39	3.51	3.30
0.900	3.46	3.35	3.43	3.56	3.35
1.000	3.42	3.31	3.39	3.53	3.31

TABLE 2.2

Bit energy to noise ratio (in dB) required to achieve a symbol error probability of $p_s = 0.1$ for $M = 32$ and diversity level $L = 5$.

ρ	Square-Law	Linear	Square-Root	Magnitude-Law	Optimum
0.640	-5.80	-5.70	-5.49	-5.56	-5.82
0.650	-2.25	-2.21	-2.02	-2.06	-2.28
0.700	1.53	1.49	1.64	1.65	1.45
0.800	3.26	3.17	3.31	3.35	3.16
0.900	3.84	3.74	3.87	3.92	3.73
1.000	4.10	3.98	4.11	4.17	3.98

linear combining is nearly optimum at large signal to noise ratios, and we do not expect that the maximum E_b/N_I required over the range of ρ can be made much smaller. It is at the crossover in performance of linear combining and square-law combining where the optimum combining technique may show improvement. However, as might be expected, optimum combining does not perform significantly better than linear combining or square-law combining at other values of ρ. The differences among the five combining schemes discussed are only a few tenths of a dB.

Table 2.2 gives a comparison of the five different diversity combining schemes for $L=5$. The improvement of optimum combining over linear and square-law combining at the crossover in performance of these suboptimum schemes is not very significant. The difference in performance is still less than 0.1dB. In view of the examples presented, we conclude that for systems with perfect side information, the diversity combining techniques discussed all perform nearly the same. It is the implementation considerations that are important in making the decision regarding which diversity combining technique to employ.

2.5 Effects of Quiescent Noise

The results presented in this chapter are valid, given that the quiescent noise level is negligible compared with the partial-band interference. For moderate levels of quiescent interference, the performance of the diversity system will be degraded from that given, but the relative performance among the diversity combining techniques would not vary greatly. For quiescent noise levels comparable to the partial-band interference levels, the results would not hold. However, if the diversity system is subject to high levels of quiescent interference, the reliability of the side information may be questionable and a different approach should be used. This is one of the reasons for looking at alternative diversity combining techniques that do not require side information. Another reason is that extracting side information from external sources can be difficult in practice. The remainder of the thesis is devoted to analyzing systems that do not depend on side information, and that have acceptable performance in the presence of partial-band interference. The performance of a system with side information is looked upon as a goal to try to achieve for systems without side information.

CHAPTER 3

CLIPPED LINEAR COMBINING

Recall from Section 2.1, that ρ^* is defined as the largest number such that a given error probability can be achieved for any interference duty factor less than ρ^* regardless of the interference power and distribution. When the receiver has no side information and combines all diversity receptions of a given symbol, the use of the diversity techniques discussed in Chapter 2 actually decreases the value of ρ^*. A symbol error is possible even if only one diversity reception has interference. Given the symbol error probability p_s and diversity level L, a system with no side information and no coding has $\rho^* = 1-(1-p_s)^{1/L}$. This small value for ρ^* indicates that the diversity combining techniques of Chapter 2, used in receivers with no side information, have practically no narrowband interference rejection capability. They perform poorly even for small values of ρ. Because $\frac{1}{2}\rho^{-1}N_I$ is large when ρ is small, it may be desirable to clip or limit the outputs of the envelope detectors. In this chapter, we analyze a system that utilizes *clipped linear combining* of diversity receptions. The role of the clipper is to constrain the effects of strong narrowband interference.

We consider the system with M-ary orthogonal signaling. In a system with clipped linear combining, each envelope detector output of each diversity reception is clipped at a level C before the diversity receptions are added. Let β denote the envelope detector output voltage due to signal only, called the signal output voltage. The clipping level is expressed as a fraction c times the signal output voltage; i. e., $C = c\,\beta$. A block diagram is shown in Figure 3.1 for the k-th branch of a diversity system that employs clipped linear combining. The quantities $\{R_{k,l} : 0 \leqslant k \leqslant M-1,\ 1 \leqslant l \leqslant L\}$ are the envelope detector outputs before they are clipped, and $\hat{x}$ denotes $\min(x, C)$ so that $\hat{x}$ is the output of the clipper when x is the input. The parameter Z_k is the combination of the diversity receptions. Given the diversity level L and the performance level p_s, ρ^* can be calculated as a function of the *relative* cutoff value c.

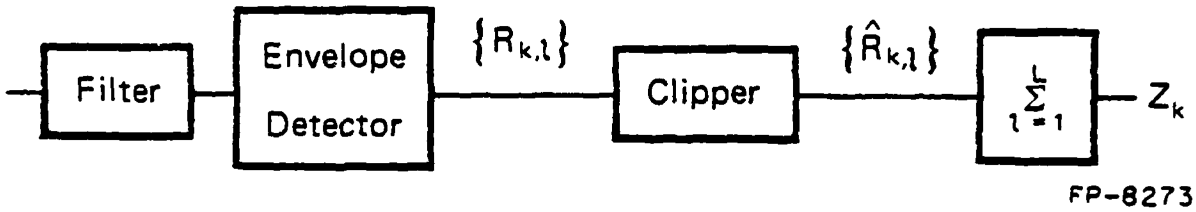

Figure 3.1. Block diagram for the k-th branch of a diversity system with clipped linear combining

3.1 Calculation of ρ^*

To calculate ρ^* for this diversity scheme, we must consider general pulsed interference of arbitrary power and statistical distribution. The worst situation for the l-th diversity reception, given that the symbol 0 is sent, is $R_{0,l}=0$ and $R_{k,l} \geqslant c\beta$ for $k>0$. At diversity level $L=1$, ρ^* is equal to the symbol error probability p_s for any value of c; interference must be present on at least p_s of the diversity receptions for there to be that fraction of errors. For $L=2$ and for c greater than one, an error can occur even if only one diversity reception has interference present, while for c less than or equal to one, an M-fold tie can occur. Because a tie results in an error with probability $(M-1)/M$, which is approximately 1 if M is large, and because we consider the worst-case situation to calculate ρ^*, we assume that ties always result in errors in this section. Clearly, for $L=2$, an error occurs if both diversity receptions have interference present. We conclude that $\rho^* = 1-\sqrt{1-p_s}$.

For diversity level $L=3$, there are two values of ρ^* depending on the value of c. For $c \geqslant 2$, an error can occur if any of the diversity receptions have interference, and so $\rho^*=1-(1-p_s)^{1/3}$. For $c<2$, an error can occur if 2 or more diversity receptions have interference, but an error cannot occur if only one diversity reception has interference. Thus, ρ^* is the solution of $p_s=\rho^3+3\rho^2(1-\rho)$ subject to the constraint $0 \leqslant \rho \leqslant 1$.

In general, if L is even, the function ρ^* versus c is broken into $L/2$ regions; if L is odd, the function is broken into $(L-1)/2$ regions. For a given symbol error probability, ρ^* is found by solving the equations

$$p_s = \sum_{j=\lfloor \frac{L+1}{2} \rfloor}^{L} \binom{L}{j} \rho^j (1-\rho)^{L-j} \qquad \begin{cases} 0<c<\dfrac{L+1}{L-1}, & L \text{ odd} \\[2ex] 0<c<\dfrac{L+2}{L-2}, & L \text{ even} \end{cases}$$

$$p_s = \sum_{j=\frac{L-i}{2}}^{L} \binom{L}{j} \rho^j (1-\rho)^{L-j}, \quad \frac{L+i}{L-i} \leqslant c < \frac{L+i+2}{L-i-2} \quad \begin{cases} i=1,3,\dots,L-2, & L \text{ odd} \\ i=2,4,\dots,L-2, & L \text{ even.} \end{cases} \tag{3.1}$$

subject to the constraint $0 \leqslant \rho \leqslant 1$. Table 3.1 lists the values for ρ^* as a function of c for $p_s = 0.1$ and for diversity levels $1 \leqslant L \leqslant 7$.

Another parameter we consider that characterizes narrowband interference rejection capability is the parameter $\rho_{\min}$, which is defined for Gaussian partial-band interference. The parameter $\rho_{\min}$ is the largest number such that a given error probability can be achieved in the presence of Gaussian partial-band interference of any duty factor less than $\rho_{\min}$. Because $\rho_{\min}$ depends on the signal to noise ratio as well as the symbol error probability, the diversity level, and the clipping level, it is calculated numerically for each particular example.

3.2 Clipped Linear Combining with Background Noise

We now calculate the performance of clipped linear combining in the presence of Gaussian partial-band interference. We also allow a nonzero quiescent noise level to account for thermal noise in the receiver and other wideband noise sources. As before, the partial-band interference is present on a given diversity reception with probability ρ, and with probability $1-\rho$, it is not present. In either case, the diversity reception is received in the presence of Gaussian quiescent noise. This quiescent noise has uniform power spectral density $\frac{1}{2}N_0$. Thus, on the fraction of the frequency band with interference present, the power spectral density is $\frac{1}{2}\rho^{-1}N_I + \frac{1}{2}N_0$, and on the fraction of the frequency band with interference absent, the power spectral density is $\frac{1}{2}N_0$.

The symbol error probability for the system with clipped linear combining, given that the symbol 0 is sent, may be expressed as

$$p_s = \sum_{0}^{L} \binom{L}{j} \rho^j (1-\rho)^{L-j} \left[1 - \int_0^\infty \hat{f}_0(x) \left[\int_0^x \hat{f}_k(y)\, dy \right]^{M-1} dx \right]. \tag{3.2}$$

The densities $\hat{f}_k(x)$ for $0 \leqslant k \leqslant M-1$ are the densities for the decision statistics

$$Z_0 = \sum_{l=1}^{j} \min\left(\sqrt{(X_{0,l}^{I} + \beta)^2 + (Y_{0,l}^{I})^2}\,, C\right) + \sum_{l=j+1}^{L} \min\left(\sqrt{(X_{0,l}^{N} + \beta)^2 + (Y_{0,l}^{N})^2}\,, C\right) \tag{3.3a}$$

TABLE 3.1

Values of ρ^* as a function of c for various L and for $p_s = 0.1$

Diversity, L	Relative Cutoff, c	ρ^*
1	all c	0.1000
2	all c	0.0513
3	$0 < c < 2$	0.1958
	$c \geq 2$	0.0345
4	$0 < c < 3$	0.1426
	$c \geq 3$	0.0260
5	$0 < c < \frac{3}{2}$	0.2466
	$\frac{3}{2} \leq c < 4$	0.1122
	$c \geq 4$	0.0208
6	$0 < c < 2$	0.2009
	$2 \leq c < 5$	0.0926
	$c \geq 5$	0.0174
7	$0 < c < \frac{4}{3}$	0.2786
	$\frac{4}{3} \leq c < \frac{5}{2}$	0.1696
	$\frac{5}{2} \leq c < 6$	0.0788
	$c \geq 6$	0.0149

$$Z_k = \sum_{l=1}^{j} \min(\sqrt{(X_{k,l}^I)^2 + (Y_{k,l}^I)^2}\,, C) + \sum_{l=j+1}^{L} \min(\sqrt{(X_{k,l}^N)^2 + (Y_{k,l}^N)^2}\,, C), \quad 1 \leqslant k \leqslant M-1. \tag{3.3b}$$

where β is the envelope detector output voltage due to signal only, C is the clipping level, and j is the number of diversity receptions with interference present. The random variables $\{X_{k,l}^I, Y_{k,l}^I : 0 \leqslant k \leqslant M-1, 1 \leqslant l \leqslant j\}$ are mutually independent zero-mean Gaussian random variables with variance σ_I^2, and $\{X_{k,l}^N, Y_{k,l}^N : 0 \leqslant k \leqslant M-1, j+1 \leqslant l \leqslant L\}$ are mutually independent zero-mean Gaussian random variables with variance σ_N^2. The relationship between β and σ_I is

$$\begin{aligned} v_I \triangleq \frac{\beta}{\sigma_I} &= \sqrt{2(E_s/(\rho^{-1}N_I + N_0))/L} \\ &= \sqrt{2 \log_2 M \ (E_b/(\rho^{-1}N_I + N_0))/L}\,. \end{aligned} \tag{3.4}$$

and the relationship between β and σ_N is

$$\begin{aligned} v_N \triangleq \frac{\beta}{\sigma_N} &= \sqrt{2(E_s/N_0)/L} \\ &= \sqrt{2 \log_2 M \ (E_b/N_0)/L}\,. \end{aligned} \tag{3.5}$$

If interference is present on a diversity reception, the signal to noise ratio is v_I, and if interference is absent on a diversity reception, the signal to noise ratio is v_N.

Let $\{R_{k,l}^I : 0 \leqslant k \leqslant M-1\}$, denote the envelope detector outputs before clipping given interference is present on the l-th diversity reception. Given that the symbol 0 is sent, the density of $R_{0,l}^I$ is the Rician density with parameters β and σ_I, denoted by $f_0^I(x)$. That is,

$$f_0^I(x) = x \exp\{-\frac{x^2+\beta^2}{2\sigma_I^2}\} I_0(\frac{x\beta}{\sigma_I^2}), \ x > 0. \tag{3.6}$$

The density of the other $M-1$ envelope detector outputs is the Rayleigh density with parameter σ_I, denoted by $f_k^I(x)$. That is,

$$f_k^I(x) = x \exp\{-\frac{x^2}{2\sigma_I^2}\}, \; x > 0. \tag{3.7}$$

Similarly, given that interference is absent on the l-th diversity reception, the envelope detector outputs before clipping are $\{R_{k,l}^N : 0 \le k \le M-1\}$. The density for $R_{0,l}^N$ is $f_0^N(x)$, and the other $M-1$ envelope detector outputs have density $f_k^N(x)$. The conditional densities $f_0^N(x)$ and $f_k^N(x)$ are found by replacing σ_I^2 by σ_N^2 in (3.6) and (3.7).

The densities in (3.6) and (3.7), and the corresponding densities for interference absent are the density functions for the situation in which $C = \infty$ (i. e., no clipper). For finite values of $C = c\beta$, the density function for the sum of j clipped envelope detector outputs for the j diversity receptions with interference present is

$$\hat{f}_k^I(x) = \sum_{l=0}^{j} \binom{j}{l} [G_k^I(C)]^{j-l} [g_k^I(x-(j-l)C)]^{(l)} \tag{3.8}$$

for each k, where

$$[g_k^I(x)]^{(1)} = f_k^I(x) p_C(x), \tag{3.9}$$

$$[g_k^I(x)]^{(0)} = \delta(x),$$

$$p_C(x) = \begin{cases} 1, & 0 \le x \le C \\ 0, & \text{otherwise,} \end{cases}$$

and, for $l > 1$, $[g_k^I(x)]^{(l)}$ is the $(l-1)$-fold self-convolution of $[g_k^I(x)]^{(1)}$. The quantity $G_k^I(C)$ is the area in the tail of the k-th non-clipped envelope detector output. That is,

$$\begin{aligned} G_k^I(C) &= 1 - F_k^I(C) \\ &= 1 - \int_0^C f_k^I(x)dx \\ &= \int_C^\infty f_k^I(x)dx, \end{aligned} \tag{3.10}$$

where $F_k^I(x)$ is the distribution function of the envelope detector outputs before they are clipped. In other words, $G_k^I(C)$ is the conditional probability that an envelope detector output is above the clipping level and is clipped to level C, given that interference is present. In particular, for $k \neq 0$,

$$G_k^I(C) = e^{-C^2/(2\sigma_I^2)}. \tag{3.11}$$

Note that $[g_k^I(x)]^{(1)}$ is not a density function. However, $[g_k^I(x)]^{(1)} + \delta(x-C)G_k^I(C)$ is the conditional density function of the k-th clipped envelope detector output given that interference is present.

Similarly, the density function for the sum of $L-j$ clipped envelope detector outputs given that interference is absent is

$$\hat{f}_k^N(x) = \sum_{l=0}^{L-j} \binom{L-j}{l} [G_k^N(C)]^{L-j-l} [g_k^N(x-(L-j-l)C)]^{(L-j-l)}. \tag{3.12}$$

The functions $[g_k^N(x)]^{(1)}$ and $G_k^N(C)$ are found by replacing σ_I by σ_N in (3.9)-(3.11). The density function for the sum of the L diversity receptions corresponding to the k-th symbol is $\hat{f}_k^i(x)$ convolved with $\hat{f}_k^N(x)$.

To calculate the probability of error in (3.2), we can normalize the densities in (3.8) and (3.12) with respect to σ_I or σ_N. That is, we can let $x' = x/\sigma_I$ or we can let $x' = x/\sigma_N$ in both (3.8) and (3.12). For example, upon normalizing with respect to σ_N, the normalized clipping level becomes cv_N and the ratio v_I/v_N enters into the density function in (3.8). The result of this normalization is that we never have to specify σ_I, σ_N, or β explicitly. The symbol error probability depends upon L, M, c, E_s/N_0, and E_s/N_I.

3.3 The Clipper Phenomenon

For diversity levels greater than 1, there are many situations, depending on the value of C, when $N_I = \infty$ *is not the worst-case interference power*. To illustrate how this can be possible, consider a binary system ($M=2$) and assume $C>0$. Let $\{\hat{R}_{k,l}^N : k=0,1,\ 1 \leq l \leq L\}$ denote the envelope detector outputs after clipping for a diversity reception with interference absent and $\{\hat{R}_{k,l}^I : k=0,1,\ 1 \leq l \leq L\}$ denote the envelope detector outputs after clipping for a diversity reception with interference present. Given that the symbol 0 is sent, the bit error probability can be expressed as

$$p_b = \sum_{j=0}^{L} \binom{L}{j} \rho^j (1-\rho)^{L-j} p_{b,j} \tag{3.13}$$

where

$$p_{b,j} = P\left(\sum_{l=1}^{L-j} \hat{R}_{1,l}^N + \sum_{l=L-j+1}^{L} \hat{R}_{1,l}^I \geq \sum_{l=1}^{L-j} \hat{R}_{0,l}^N + \sum_{l=L-j+1}^{L} \hat{R}_{0,l}^I\right). \tag{3.14}$$

is the conditional probability of error given that j diversity receptions have interference present. For a fixed ρ, as $N_I \to 0$, the power spectral density goes to $\frac{1}{2}N_0$ (whether interference is present or absent), and

$$p_{b,j} = P\left(\sum_{l=1}^{L} \hat{R}_{1,l}^N \geq \sum_{l=1}^{L} \hat{R}_{0,l}^N\right). \tag{3.15}$$

for each j. Substituting (3.15) into (3.13) gives (3.13). That is, for $N_I = 0$, p_b is the same as (3.15), independent of ρ and j. This is the probability of error for a binary system with clipped linear combining and diversity L in full-band additive white Gaussian noise.

Now consider what happens as $N_I \to \infty$. Every diversity reception with interference present is clipped at C. That is, an upper bound on $F_0^I(C) = 1 - G_0^I(C)$ is

$$F_0^I(C) \leq e^{-\beta^2/(2\sigma_I^2)} - e^{-(C-\beta)^2/(2\sigma_I^2)} + \frac{\beta}{\sigma_I}\sqrt{2\pi}\left[Q\left(\frac{\beta - C}{\sigma_I}\right) - Q\left(\frac{\beta}{\sigma_I}\right)\right]. \tag{3.16}$$

which goes to zero as σ_I^2 goes to infinity. The bound in (3.16) is found by using $I_0(x) \leq e^{x}$. Thus, with probability 1, $\hat{R}_{0,l}^I = C$ given that $N_I = \infty$. Also note from (3.11) that $G_1^I(C) \to 1$ if $\sigma_I^2 \to \infty$. Therefore, for $N_I \to \infty$, we have that

$$\begin{aligned} p_{b,j} &= P\left(\sum_{l=1}^{L-j} \hat{R}_{1,l}^N + jC \geq \sum_{l=1}^{L-j} \hat{R}_{0,l}^N + jC\right) \\ &= P\left(\sum_{l=1}^{L-j} \hat{R}_{1,l}^N \geq \sum_{l=1}^{L-j} \hat{R}_{0,l}^N\right). \end{aligned} \tag{3.17}$$

Thus, for a fixed ρ, interference of infinite power applied to a given diversity reception essentially erases that diversity reception. By comparing (3.14) for finite interference power to

(3.17) for infinite interference power, we note that there may be situations when (3.17) is greater than (3.14). That is, a jammer may do worse than canceling out a diversity reception. However, it is difficult to show this analytically by using (3.14) and (3.17). Thus, we illustrate this by reverting to the situation in which there is no quiescent noise (i. e., the limiting case in which $N_0 \to 0$).

The conditional bit error probability given that j diversity receptions have interference present in (3.14) becomes

$$p_{b,j} = P\Big(\sum_{l=L-j+1}^{L} \hat{R}_{1,l}^{I} \geqslant \sum_{l=L-j+1}^{L} \hat{R}_{0,l}^{I} + (L-j)\hat{\beta}\Big) \tag{3.18}$$

as $N_0 \to 0$, where $\hat{\beta}$ is the signal output voltage after clipping. We can write (3.18) as

$$p_{b,j} = \begin{cases} \displaystyle\sum_{n=1}^{j} \binom{j}{n} P\Big(\sum_{l=1}^{j} \hat{R}_{1,l}^{I} > \sum_{l=1}^{j} \hat{R}_{0,l}^{I} + (L-j)\hat{\beta} \,|\, E_n\Big) P(E_n), & 1 \leqslant j < L \\ \displaystyle\sum_{n=1}^{L} \binom{L}{n} P\Big(\sum_{l=1}^{L} \hat{R}_{1,l}^{I} > \sum_{l=1}^{L} \hat{R}_{0,l}^{I} \,|\, E_n\Big) P(E_n) + \frac{1}{2} T(C), & j = L. \end{cases} \tag{3.19}$$

where E_n is the event

$$E_n = \begin{cases} \displaystyle\bigcap_{l=1}^{j} \{R_{0,l}^{I} \geqslant C\}, & n = 0 \\ \displaystyle\Big[\bigcap_{l=1}^{n} \{R_{0,l}^{I} < C\}\Big] \bigcap \Big[\bigcap_{l=n+1}^{j} \{R_{0,l}^{I} \geqslant C\}\Big], & 1 \leqslant n \leqslant j-1 \\ \displaystyle\bigcap_{l=1}^{j} \{R_{0,l}^{I} < C\}, & n = j. \end{cases}$$

In (3.19), $T(C)$ is defined by

$$T(C) = [G_0^I(C)]^L \, [G_1^I(C)]^L . \tag{3.20}$$

which is the probability of a tie given that all diversity receptions have interference present; that is, (3.20) is the probability that all the envelope detector outputs are clipped at level C

given that $j=L$. We assume that such ties are correctly resolved with probability $\frac{1}{2}$. The probability of the event E_n is

$$P(E_n)=[F_0^l(C)]^n[G_0^l(C)]^{j-n}. \tag{3.21}$$

Notice that

$$P(\sum_{i=1}^{j}\hat{R}_{1,i}^{l}>\sum_{i=1}^{j}\hat{R}_{0,i}^{l}+(L-j)\hat{\beta}\mid E_0)\equiv 0. \tag{3.22}$$

It can be shown that $P(E_n)\to 0$ for $n>0$ as $\sigma_I^2\to\infty$. From (3.16), we have that $F_0^l(C)\to 0$ as $\sigma_I^2\to\infty$. From (3.21), $F_0^l(C)\to 0$ implies that $P(E_n)\to 0$ for $n>0$. Also, $F_0^l(C)\to 0$ implies that $G_0^l(C)\to 1$ as $\sigma_I^2\to\infty$. Because $G_1^l(C)\to 1$ as $\sigma_I^2\to\infty$, we conclude that $\lim_{\sigma_I^2\to\infty}T(C)=1$. In conjunction with (3.22), (3.19), and (3.13), we have that $p_b\to\rho^L/2$ as $\sigma_I^2\to\infty$. The interesting thing to note here is that $\lim_{\sigma_I^2\to\infty}p_{b,j}=0$ for $j<L$. That is, the conditional bit error probability given that less than L diversity receptions have interference present, goes to 0 as the interference power goes to ∞. If interference is present on all L diversity receptions, a tie is the worst thing that results for infinite interference power.

Next we show that, for each j, $p_{b,j}$ in (3.19) goes to 0 as $\sigma_I^2\to 0$. First, we claim that $\lim_{\sigma_I^2\to 0}p_{b,L}=0$. Note that we can use the union bound on $p_{b,L}$:

$$\begin{aligned}p_{b,L}&=P(\sum_{l=1}^{L}\hat{R}_{0,l}^{l}\leqslant\sum_{l=1}^{L}\hat{R}_{1,l}^{l})\leqslant P(\bigcup_{l=1}^{L}\{\hat{R}_{0,l}^{l}\leqslant\hat{R}_{1,l}^{l}\})\\&\leqslant\sum_{l=1}^{L}P(\hat{R}_{0,l}^{l}\leqslant\hat{R}_{1,l}^{l}).\end{aligned}$$

Since $\{\hat{R}_{0,l}^{l}\leqslant\hat{R}_{1,l}^{l}\}\subset\{R_{0,l}^{l}\leqslant R_{1,l}^{l}\}\bigcup\{R_{1,l}^{l}>C\}$, we can write

$$\begin{aligned}P(\hat{R}_{0,l}^{l}\leqslant\hat{R}_{1,l}^{l})&\leqslant P(R_{0,l}^{l}\leqslant R_{1,l}^{l})+P(R_{1,l}^{l}>C)\\&=\frac{1}{2}e^{-\beta/(4\sigma_l^2)}+e^{-C^2/(2\sigma_l^2)}\end{aligned}$$

which goes to zero as σ_l^2 goes to zero. Similarly, $p_{b,j}\to 0$, as $\sigma_I^2\to 0$ for $j<L$. Thus, the bit

error probability given that j diversity receptions have interference present for $j < L$, is non-zero for finite σ_I^2, and tends to 0 both as $\sigma_I^2 \to 0$ and as $\sigma_I^2 \to \infty$. The terms $p_{b,j}$ for $j < L$ are not monotonic in σ_I.

Our numerical examples show that in many situations these non-monotonic components of p_b in (3.13) result in the existence of a worst-case interference power other than $N_I = \infty$. For $M=2$, $L=2$, and $N_0=0$, we present an analytical example. We know that for $L=2$,

$$p_b = 2\rho(1-\rho)p_{b,1} + \rho^2 p_{b,2} \rightarrow \rho^2/2$$

as $E_b/N_I \to 0$. For fixed ρ and $C > \beta$, we can find values of $E_b/N_I > 0$ such that $p_{b,1} > \frac{\rho}{4(1-\rho)}$, which implies that $p_b > \rho^2/2$. To show this analytically, we first find a lower bound on $p_{b,1}$ for $L=2$ and $C > \beta$. Observe that

$$p_{b,1} = P(\hat{R}_1^I > \hat{R}_0^I + \beta) = P(\{R_1^I > R_0^I + \beta\} \bigcap \{R_0^I < C - \beta\})$$

because

$$P(\{\hat{R}_1^I > \hat{R}_0^I + \beta\} \bigcap \{R_0^I > C - \beta\}) = 0$$

and

$$\{\hat{R}_1^I > \hat{R}_0^I + \beta\} \bigcap \{R_0^I < C - \beta\} = \{R_1^I > R_0^I + \beta\} \bigcap \{R_0^I < C - \beta\}.$$

Thus, we may write

$$p_{b,1} = \int_0^{C-\beta} f_0^I(x) G_1^I(x+\beta)\,dx \geqslant F_0^I(C-\beta) G_1^I(C).$$

It follows from $I_0(x) \geqslant 1$ that

$$p_{b,1} \geqslant e^{-(C^2+\beta^2)/(2\sigma_I^2)}[1 - e^{-(C-\beta)^2/(2\sigma_I^2)}],$$

which for $C = c\beta$ and $\beta/\sigma_I = \sqrt{(E_b/N_I)\rho}$ gives

$$p_{b,1} \geqslant e^{-(1+c^2)(E_b/N_I)\rho/2} - e^{-(c^2-c+1)(E_b/N_I)\rho}.$$

If $c=2$, $\rho=0.1$, and $E_b/N_I=5$, then

$$p_{b,1} \geq 0.0633 > \frac{\rho}{4(1-\rho)} = 0.028.$$

Hence, we have an example for which finite interference power gives a larger error probability than infinite interference power.

This example for $N_0=0$ implies the existence of examples for $N_0>0$. To see this, let $p_b(N_I, N_0)$ denote the bit error probability as a function of N_I and N_0. We have an example for which $p_b(N_I, 0) > p_b(\infty, 0)$. Because $p_b(N_I, N_0)$ is a continuous function of N_0, there exists an $\epsilon>0$ such that for $N_0 \leq \epsilon$, $p_b(N_I, N_0) > p_b(\infty, N_0)$. That is, the clipper phenomenon also occurs for some range of values of $0 \leq N_0 \leq \epsilon$.

Figure 3.2 is a plot of the dominant components of the symbol error probability p_s, showing that $p_{s,j}$ is not monotonic with respect to E_b/N_I for $j<L$. The conditional error probability given that all diversity receptions have interference present, $p_{s,L}$, decreases monotonically as E_b/N_I increases. Let $c_j = \binom{L}{j}\rho^j(1-\rho)^{1-j}$. The curves plotted are for $c_3 p_{s,3}$, $c_4 p_{s,4}$, $c_5 p_{s,5}$, and p_s versus E_b/N_I for the system with $M=32$, $L=5$, and E_b/N_0=18dB. The probabilities $p_{s,0}$, $p_{s,1}$, and $p_{s,2}$ are small compared to the other components of p_s. The non-monotonicity of $c_3 p_{s,3}$ and $c_4 p_{s,4}$ affects the sum in (3.13) enough so that p_s is not monotonic in E_b/N_I.

In Figures 3.3 and 3.4, we investigate the problem of finding the situations where the clipper phenomenon is most prevalent. From the figures, we see that the bit energy to noise ratio for the maximum symbol error probability increases as ρ decreases; i. e., the worst-case N_I decreases with ρ. For $\rho=1$, p_s versus E_b/N_I is monotone decreasing--the worst-case N_I is $N_I=\infty$. That is, at $\rho=1$, the phenomenon does not occur. It is at smaller values of ρ and larger values of L where the phenomenon occurs. Also, as E_b/N_0 decreases, the phenomenon is not apparent, and for small E_b/N_0 and large ρ, the phenomenon goes away completely.

Due to the non-monotonicity of p_s versus E_b/N_I, as shown in these figures, there are two solutions for E_b/N_I for some values of p_s. For large values of E_b/N_0, some prediction can be

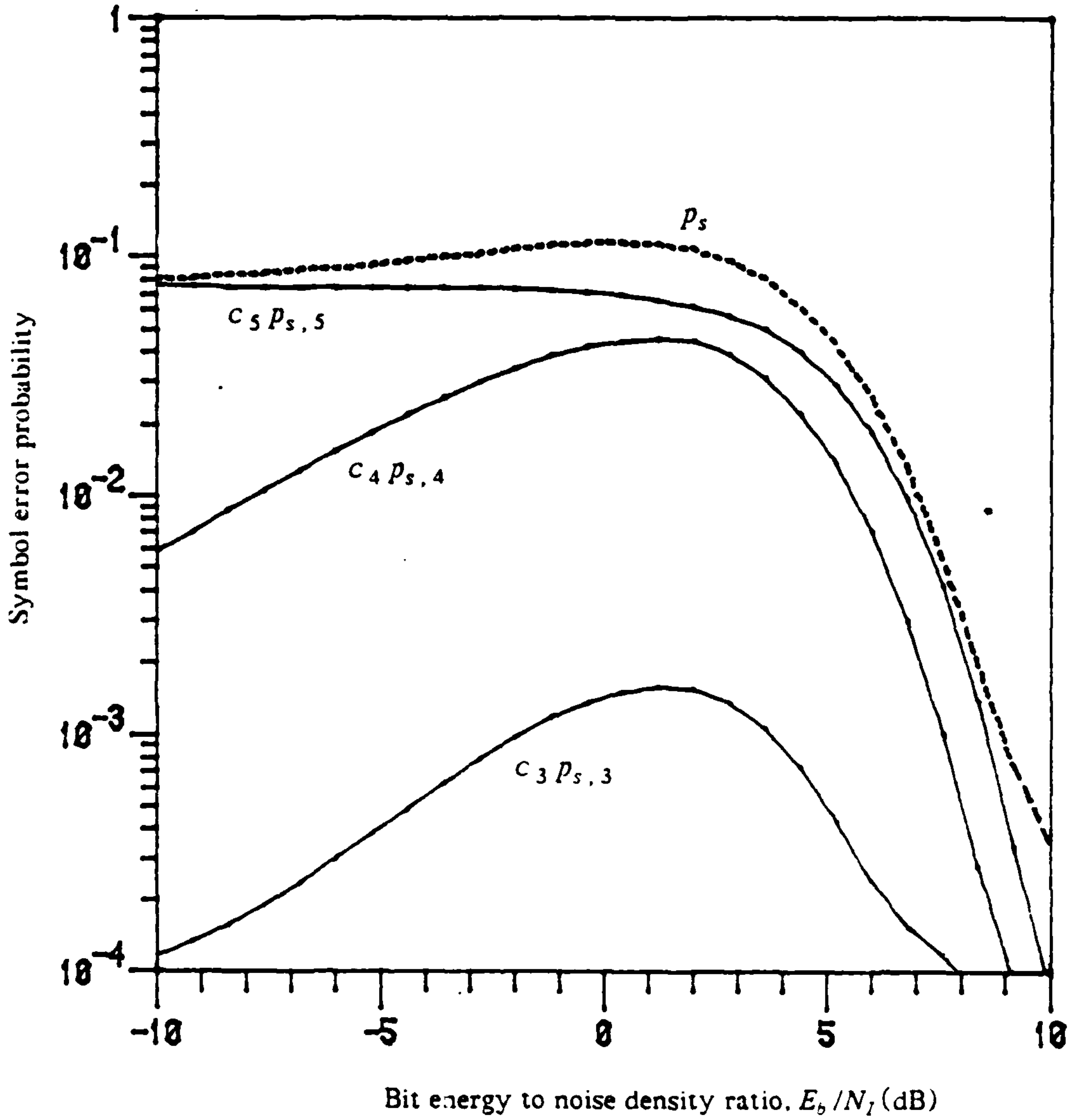

Figure 3.2. Symbol error probability versus bit energy to noise density ratio for $M=32$, $L=5$, E_b/N_0=18dB, and ρ=0.6

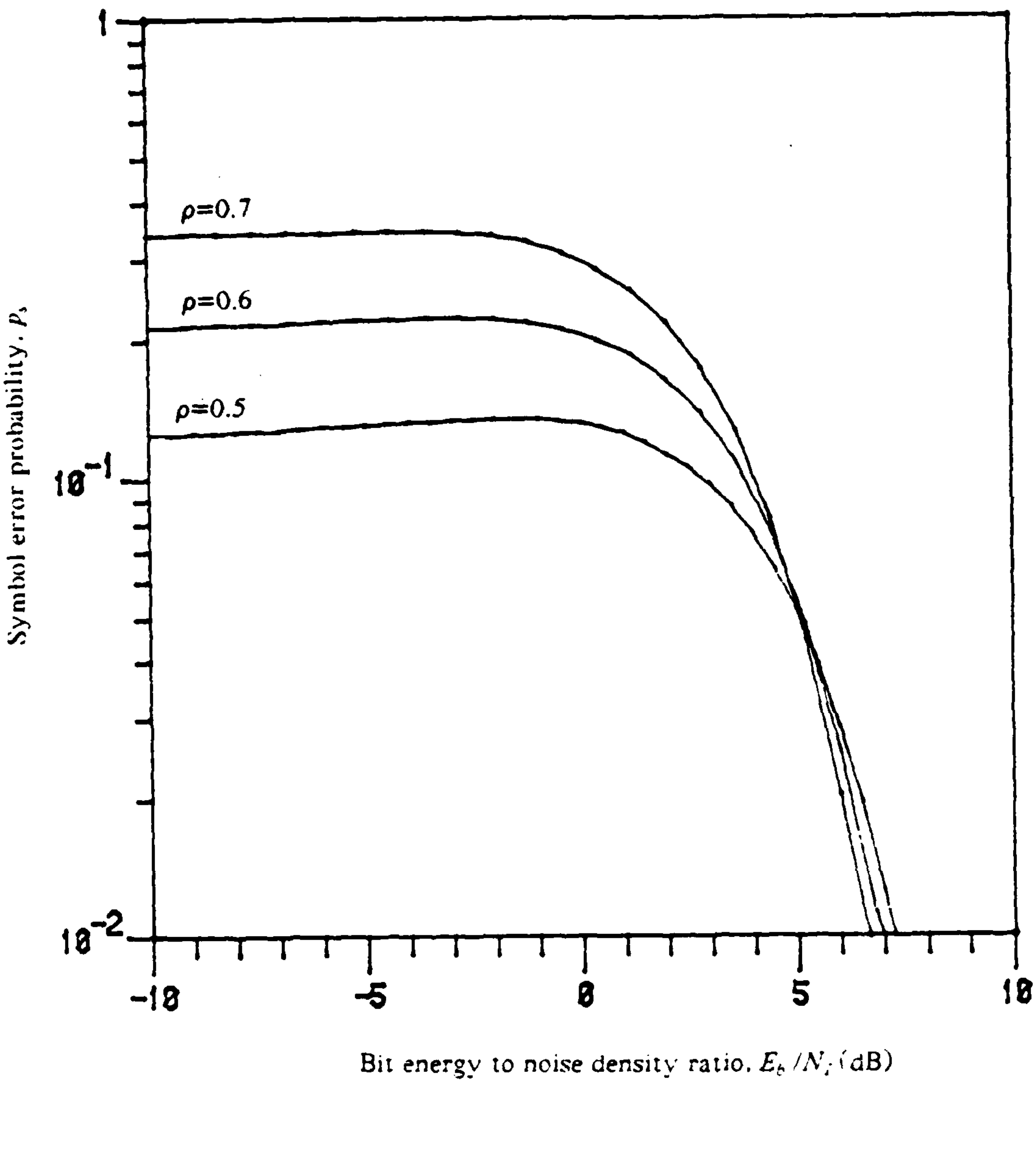

Figure 3.3. Symbol error probability versus bit energy to noise density ratio for $M=32$, $L=3$, E_b/N_0=18dB, and ρ=0.5, 0.6, 0.7

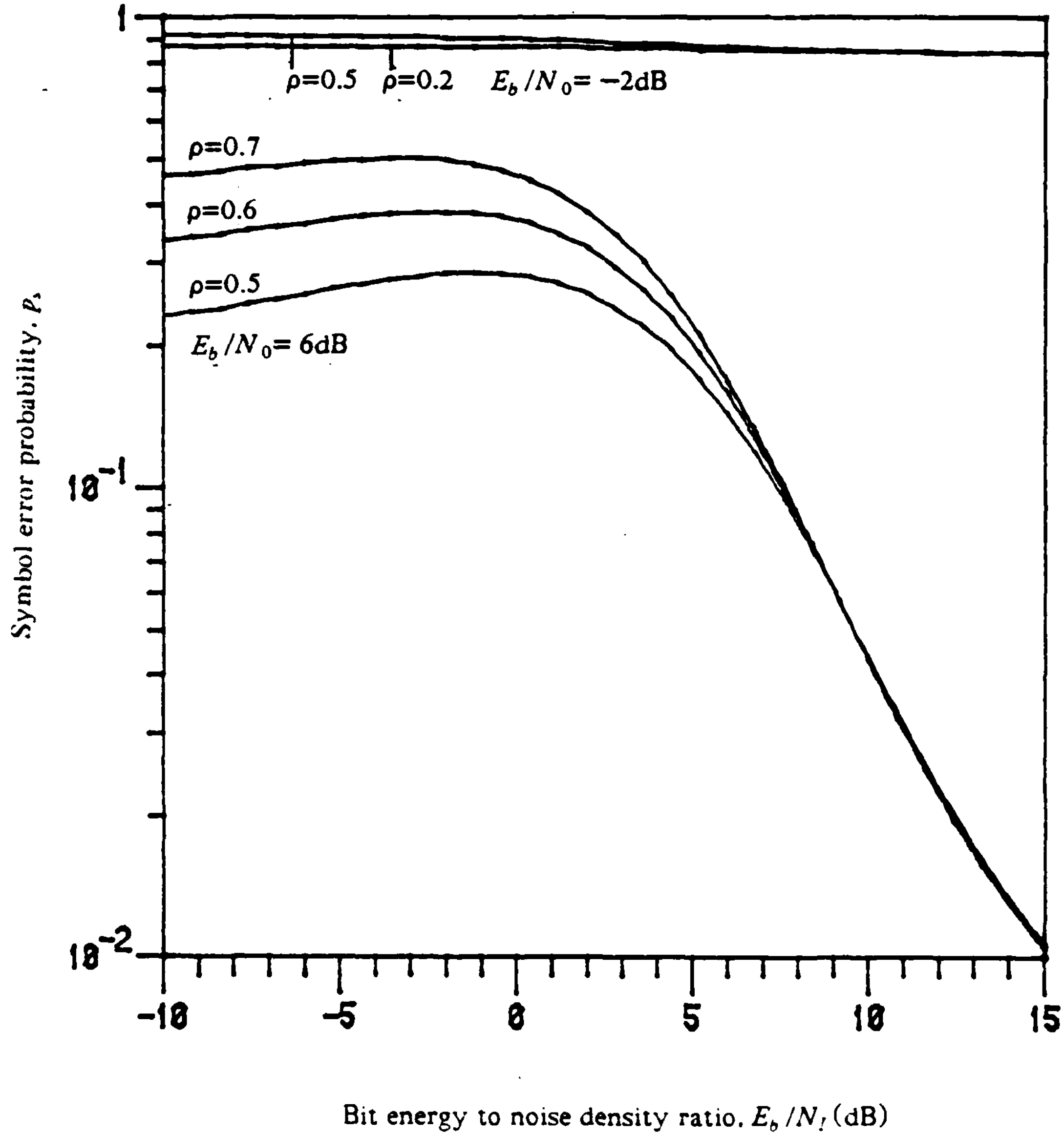

Figure 3.4. Symbol error probability versus bit energy to noise density ratio for $M=32$, $L=3$, and $\rho=0.5, 0.6, 0.7$ for E_b/N_0=6dB, and $\rho=0.2, 0.5$ for E_b/N_0=−2dB made as to when two solutions for E_b/N_I occur. That is. for large E_b/N_0 we know that $p_s \to \rho^L$ as $N_I \to \infty$. $p_s > \rho^L$ for a range of N_I. and $p_s < \rho^L$ for smaller N_I. Thus. we have that for large values of signal to quiescent noise ratios. there is one solution for E_b/N_I if $\rho > p_s^{1/L}$. two solutions for a small region of $\rho \leqslant p_s^{1/L}$. and no solutions for smaller ρ; p_s can be obtained in that region regardless of the signal to noise ratio. For smaller values of signal to quiescent noise ratios. the clipper phenomenon still occurs. However. it is difficult to predict the region of ρ where two solutions for E_b/N_I exist. because (3.15) and (3.17) cannot be solved analytically for $L \geqslant 2$.

3.4 Numerical Results

In Figure 3.5. we demonstrate the sensitivity of clipped linear combining to the clipping level for Gaussian partial-band interference. The figure shows curves of E_b/N_I versus ρ for $M=32$ and a symbol error probability of 0.1. Suppose that the desired clipping level is equal to the signal output voltage β. but due to inexact measurements at the receiver caused by the communications channel. the clipping level varies between 3dB above and below β. The value of $\rho_{\min}$ for each of the curves shown is such that for $\rho < \rho_{\min}$. a symbol error probability of 0.1 is achieved regardless of the value of E_b/N_I. For $C=\beta$. $\rho_{\min}$ is approximately 0.449. Notice that this value of $\rho_{\min}$ is much better than the ρ^* value predicted in Table 3.1. If the clipping level is set above the signal output voltage. $\rho_{\min}$ decreases showing that some of the narrowband interference rejection capability is lost; e. g., for $C=1.41\beta$. $\rho_{\min} \approx 0.350$. If the clipping level is below the signal output voltage. then $\rho_{\min}$ increases; e. g., for $C=0.708\beta$. $\rho_{\min} \approx 0.456$. However. the maximum signal to noise ratio also increases. In spite of the deviation of the clipping level from the desired value. clipped linear combining provides narrowband interference rejection capability that linear combining alone cannot.

Also included in Figure 3.5 is the performance of the system with linear combining with perfect side information (no quiescent noise). For this system. $\rho_{\min} \approx 0.475$ and $\rho^* \approx 0.464$. The performance of the system with linear combining with perfect side information is a lower

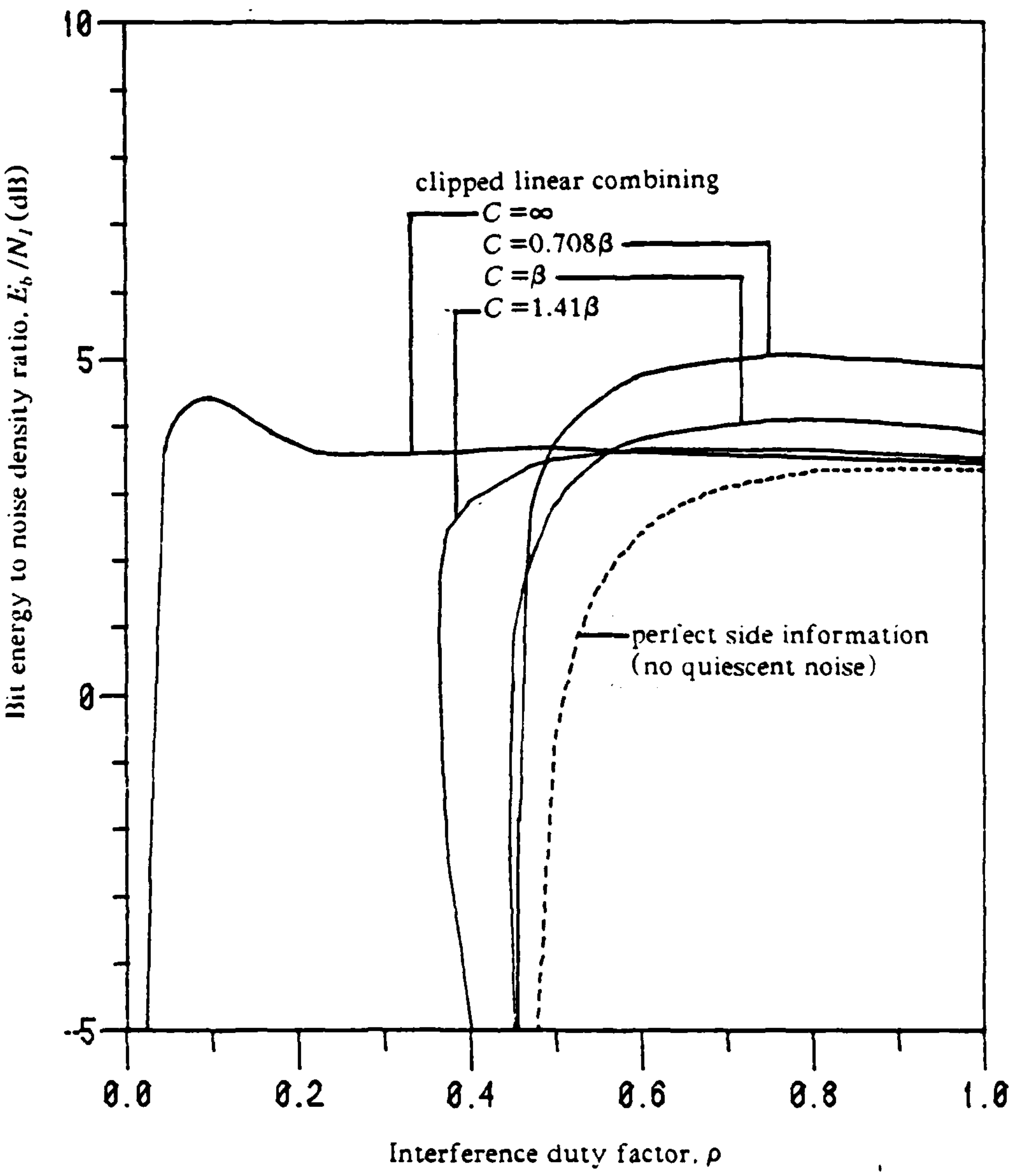

Figure 3.5. Bit energy to noise density ratio versus interference duty factor for $M = 32$, $\rho_s = 0.1$, $L = 3$, $E_b/N_0 = 18$dB, and various clipping levels C for clipped linear combining and for linear combining with perfect side information (no quiescent noise) bound on the performance of clipped linear combining. Clipped linear combining is nearly as good as linear combining with perfect side information. However, just as it may be impractical for a receiver to extract perfect side information, it may be impractical to implement clipped linear combining if there is more than a 3dB deviation in the signal output voltage.

In Figure 3.6, the effect of increasing the diversity level is shown. The curves are for diversity levels 1 through 6, clipping level $C=\beta$, $p_s=0.1$, $M=32$, and E_b/N_0=18.0dB. Narrowband interference rejection becomes better as L increases, but because of noncoherent combining losses, the system performance becomes worse for large ρ.

The sensitivity of the performance to the quiescent noise level is presented in Figure 3.7. The curves are for signal to quiescent noise ratios E_b/N_0 of 6dB, 12dB, 18dB and ∞, for the system with $L=3$, $M=32$, $C=\beta$, and $p_s=0.1$ The performance curves for E_b/N_0=18dB and $E_b/N_0=\infty$ are nearly the same.

In most of the examples in this thesis, we choose a value of symbol error probability p_s that is large (e. g., $p_s=0.1$), because we are assuming that coding will be employed in the system. With $p_s=0.1$, bit error probabilities on the order of 10^{-4} to 10^{-5} are readily achievable with coding [2]. For example, $p_s=0.1$ corresponds to a bit error probability of $4.95 \cdot 10^{-4}$ for a system using an extended (32, 16) Reed-Solomon (R-S) code for error correction. For systems using a code of rate r, the signal to noise ratios for interference present and interference absent are

$$v_I = \sqrt{2\, r\, \log_2 M\, (E_b/(\rho^{-1}N_I+N_0))/L}$$

and

$$v_N = \sqrt{2\, r\, \log_2 M\, (E_b/N_0)/L}\,.$$

Suppose coding is *not* to be employed in the system and we wish to achieve a bit error probability of $4.95 \cdot 10^{-4}$. An example of this is shown in Figure 3.8. The uncoded system requires a symbol error probability of $9.59 \cdot 10^{-4}$ for the desired bit error probability. The

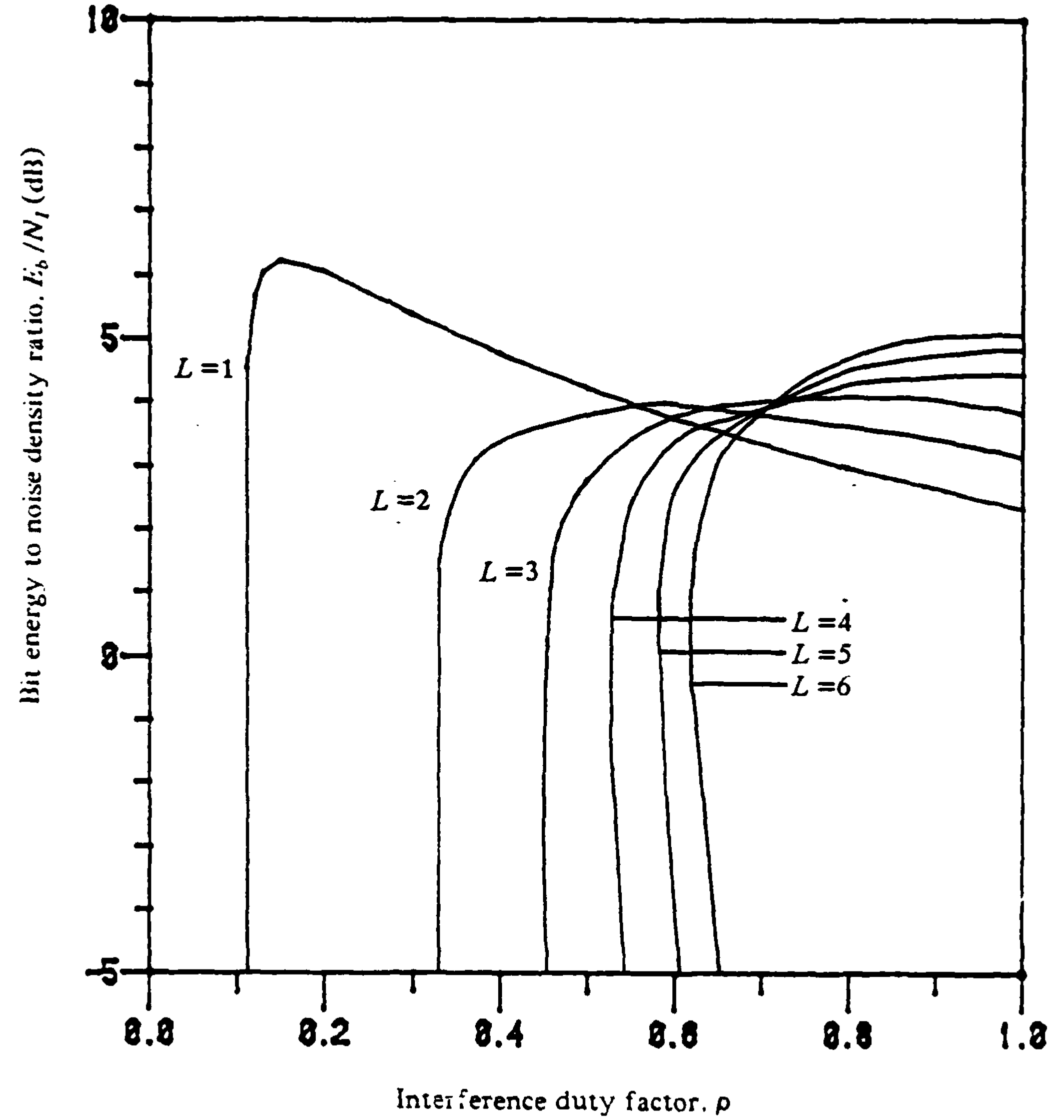

Figure 3.6. Bit energy to noise density ratio versus interference duty factor for $M=32$, $p_s=0.1$, $C=\beta$, $E_b/N_0=18$dB, and diversity levels 1 through 6 for clipped linear combining

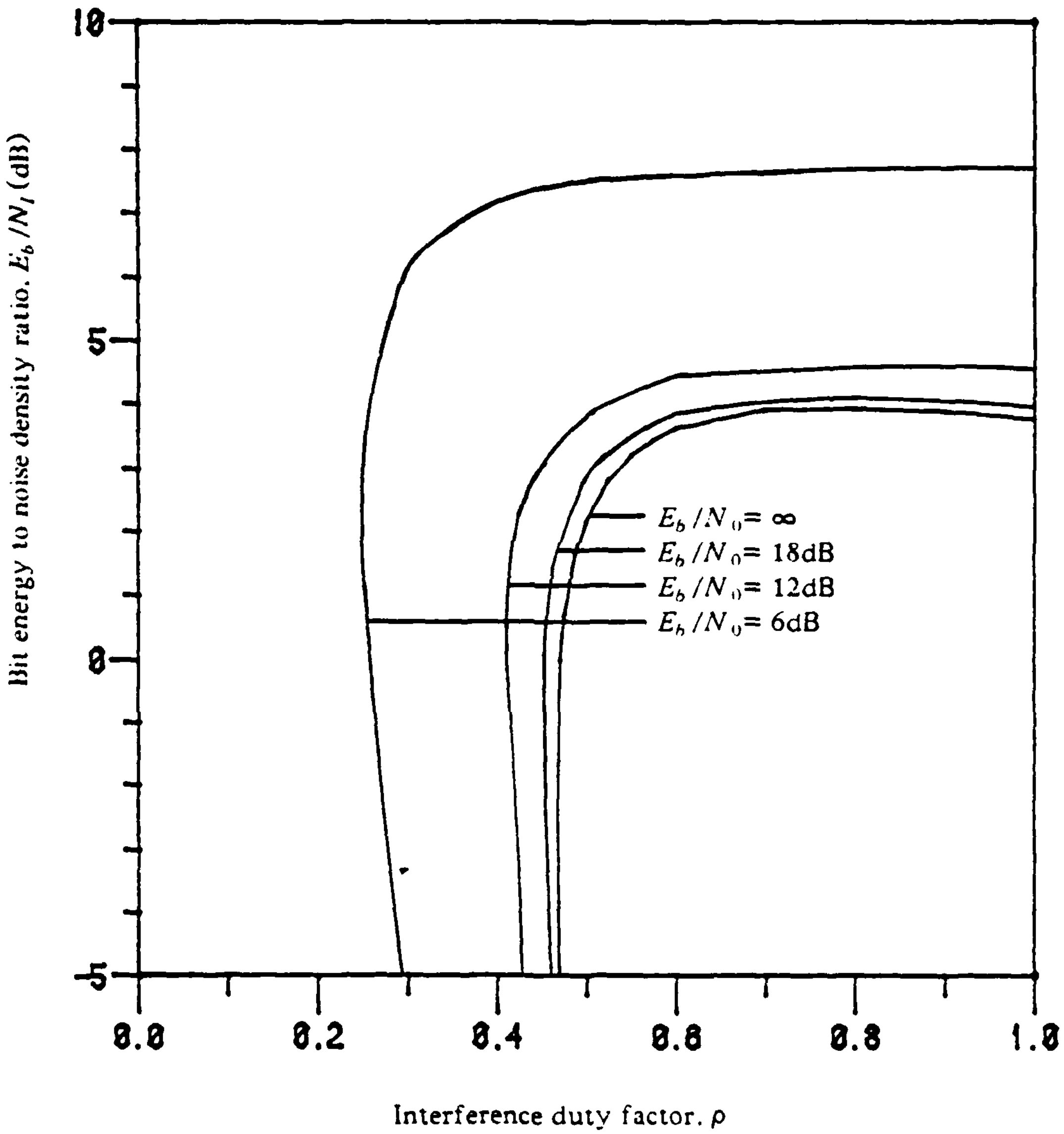

Figure 3.7. Bit energy to noise density ratio versus interference duty factor for $M=32$, $p_s=0.1$, $L=3$, $C=\beta$, and various quiescent bit energy to noise density ratios for clipped linear combining

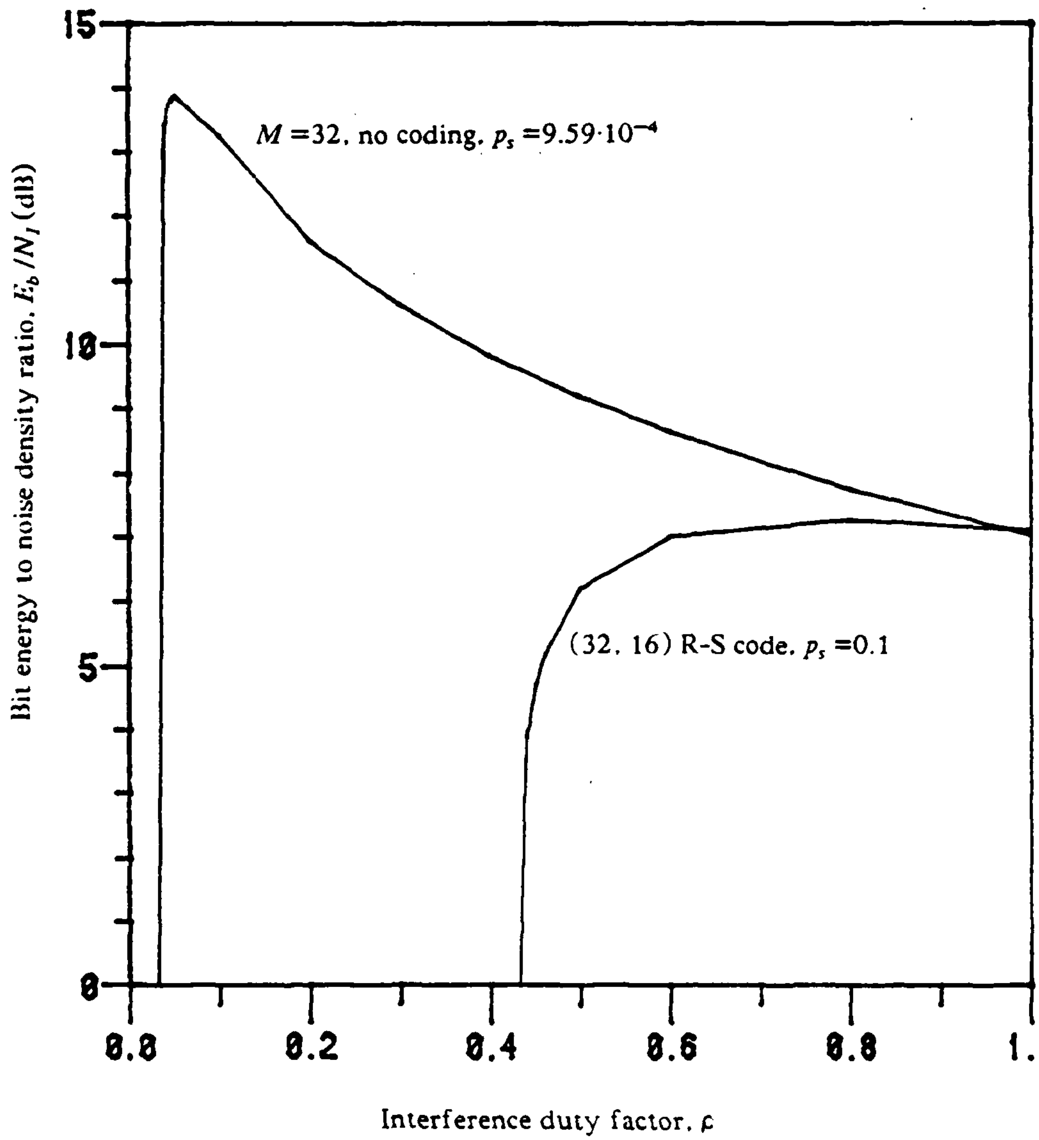

Figure 3.8. Bit energy to noise density ratio versus interference duty factor for M =32, p_b =4.95·10^{-4}, L =3, C =β, and E_b/N_0=18dB for clipped linear combining with no coding and for clipped linear combining with a (32, 16) Reed-Solomon code uncoded system has practically no narrowband interference rejection.

We next analyze tradeoffs between diversity and coding, keeping the data rate fixed. We compare a system with a low rate (n, k) R-S code with no diversity to a system with diversity and coding. We vary the parameters L and $r = k/n$, the diversity level and code rate, so the relationship that we keep fixed is $R = \frac{k \log_2 M}{nL}$. In Figure 3.9, the systems we compare are: a system with a (32, 4) R-S code and $L = 1$, a system with a (32, 8) R-S code and $L = 2$, and a system with a (32, 16) R-S code and $L = 4$. All three schemes use clipped linear combining with $C = \beta$. The desired bit error rate is $1 \cdot 10^{-4}$, and the signal to quiescent noise ratio is E_b/N_0=18dB. Undoubtedly, the system with the higher rate code and diversity is better than the system with the low rate code and no diversity in terms of both ρ_{min} and the maximum bit energy to noise ratio. Note that the scheme with $L = 4$ and a (32, 16) rate code is a concatenated code with total block length 128. We would expect a (128, 16) code to be superior to the (32, 16) code with $L = 4$, and the (128, 16) code has a higher rate. However, a block code of length 128 is more complex to decode than the length 32 code. As long as the diversity combining scheme is not complex, diversity is a simple way to increase the block length of the "overall" code.

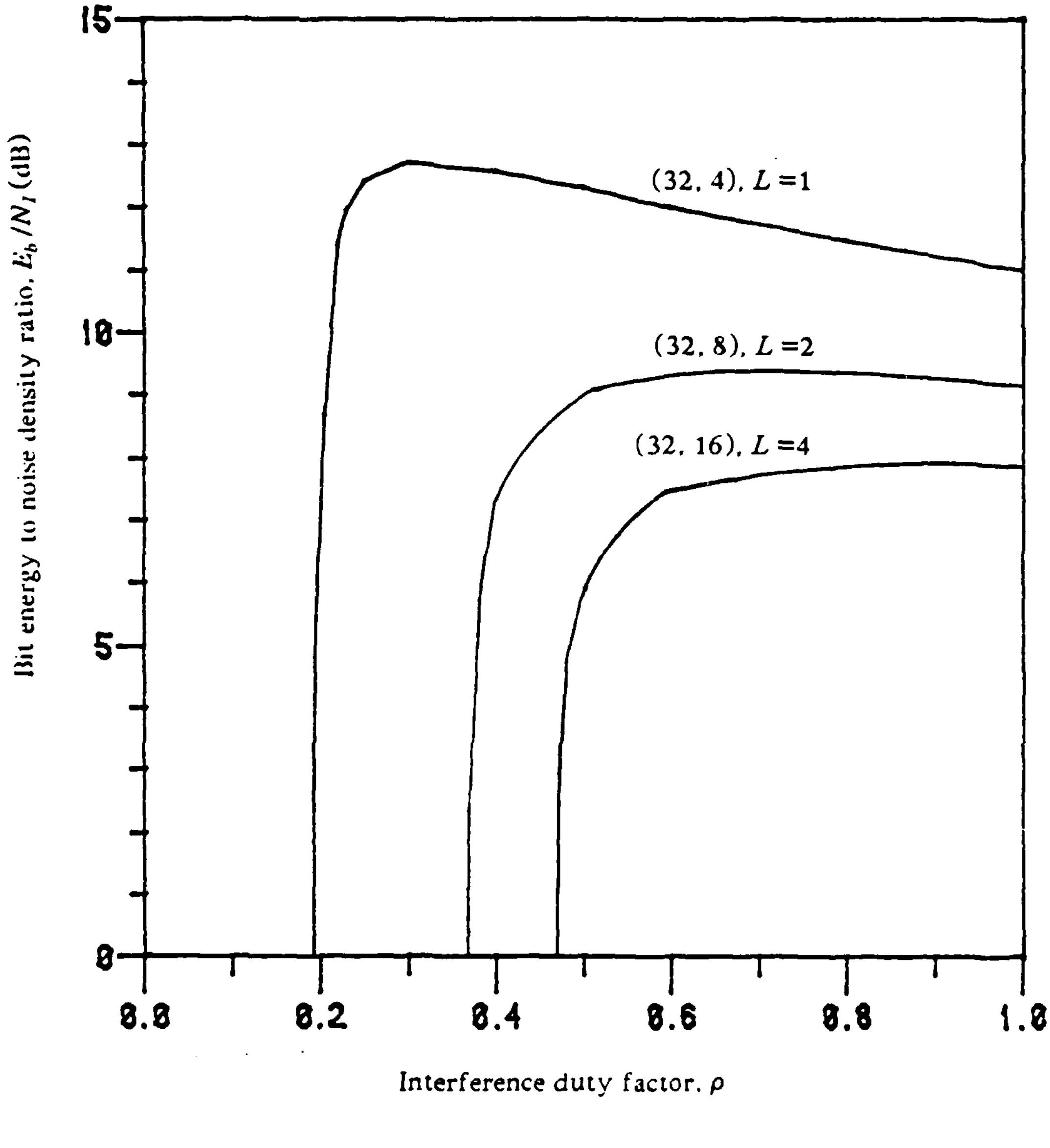

Figure 3.9. Bit energy to noise density ratio versus interference duty factor for $M=32$ $p_b=1\cdot10^{-4}$, $L=3$, $C=\beta$, and $E_b/N_0=18$dB for clipped linear combining with Reed-Solomon coding and diversity

CHAPTER 4

THE RATIO STATISTIC AND DIVERSITY COMBINING

One diversity combining technique that has been shown to be effective against partial-band interference uses a ratio statistic as a measure of the quality of a given diversity reeeption. Viterbi introduced the use of the ratio statistic in a ratio threshold test as a robust technique to use for protection against partial-band interference and tone jamming [8]. In [9] and [10], a form of the ratio threshold test is analyzed from an information theoretic point of view. The primary performance measures used in [9] and [10] are channel capacity and cutoff rate.

In this chapter, we compare several diversity combining techniques that use a ratio statistic in conjunction with diversity combining. We evaluate the error probability for each scheme proposed. We use the performance measurements discussed in Chapter 1 to determine the merit of each diversity combining technique. Thus, a diversity combining technique is judged on its narrowband interference rejection capability and on its signal to noise ratio requirement over the entire range of interference duty factors.

A desirable property of the ratio statistic is that the reliability of each diversity reception is determined independently of other diversity receptions, rather than based on a measurement such as the average received signal strength over many diversity receptions. That is, the schemes using the ratio statistic may be more "robust" for FH systems in jamming, multiple-access, and fading environments, where the received signal strength may vary from one diversity reception to the next.

We first examine the ratio threshold test applied in a system with binary orthogonal signaling. The ratio threshold test with linear combining is one diversity combining scheme considered. Another diversity combining scheme we analyze uses the ratio threshold test with majority logic decoding. The motivation for simplifying from a system with M-ary orthogonal signaling to a system with binary orthogonal signaling is that the ratio threshold test with linear combining requires extensive computation for systems with M-ary orthogonal signaling. Therefore, linear combining and majority logic decoding, used in conjunction with the ratio threshold test, are compared to each other and to clipped linear combining on the basis of their performance in a system with binary orthogonal signaling.

In a diversity scheme that employs the ratio threshold technique in a system with binary orthogonal signaling, the ratio statistic is $T_l = \max(R_{0,l}, R_{1,l})/\min(R_{0,l}, R_{1,l})$ for each $1 \leqslant l \leqslant L$. The ratio statistic for each diversity reception is compared to a threshold. The threshold, denoted by θ, is a fixed number greater than 1; θ does not depend on the signal strength at the receiver. If T_l is larger than θ, the diversity reception is accepted, and if T_l is smaller than θ, the diversity reception is rejected. The test is based on the fact that a diversity reception that has strong interference present is likely to have nearly equal energy in both envelope detector outputs; such a diversity reception is rejected by ratio threshold test.

We also discuss the ratio threshold test applied in a system with M-ary orthogonal signaling. For a system with M-ary orthogonal signaling, the ratio statistic T_l for the l-th diversity reception is the ratio of the largest envelope detector output to the second largest envelope detector output. The ratio statistic is compared to the threshold θ to determine whether or not to include the diversity reception in the decision process.

4.1 Linear Diversity Combining

In the scheme employing the ratio threshold test with linear combining for a system with binary orthogonal signaling, the ratio statistic $T_l = \max(R_{0,l}, R_{1,l})/\min(R_{0,l}, R_{1,l})$ for each $1 \leqslant l \leqslant L$ is formed for each diversity reception. If T_l is greater than a specified threshold θ, the diversity reception is accepted, and if T_l is less than θ, the diversity reception is rejected. If at least one of the diversity receptions is accepted, the accepted diversity receptions are combined. If all of the diversity receptions are rejected, then all diversity receptions are combined. The combiner adds the outputs of the envelope detectors of the selected diversity receptions, and the decision for each bit is made by comparing the sums.

Consider the situation in which there is no quiescent noise. If the threshold is equal to 1, every diversity reception is accepted by the ratio threshold test, and the diversity scheme is standard linear combining with no side information. On the other hand, as θ increases, more diversity receptions are rejected by the test. For θ equal to infinity, the only diversity receptions that are accepted are noise-free diversity receptions, since $\max(R_{0,l}, R_{1,l}) \neq 0$ and $\min(R_{0,l}, R_{1,l})=0$. There is never an error if one or more noise-free diversity receptions are received. An error can occur only if all of the diversity receptions have interference present. Thus, for large θ, the diversity scheme approximates linear combining with perfect side information.

For the situation in which there is no quiescent noise, the desirable behavior of rejecting nearly every diversity reception with interference present as θ increases is not reflected in the value of ρ^* for this scheme. We have that

$$\rho^* = 1-(1-p_b)^{1/L} \tag{4.1}$$

for the ratio threshold technique and linear combining, which is the same as ρ^* for standard linear combining with no side information. That is, considering the worst-case pulsed interference, an error can occur even if only one diversity reception has interference present. However, the value of $\rho_{\min}$ for Gaussian interference is expected to depend on θ and to be greater than ρ^*.

In the presence of quiescent noise, all of the diversity receptions are rejected for θ equal to infinity. Thus, the ratio threshold test with linear combining reduces to standard linear combining (no side information) as θ approaches 1 or as θ approaches ∞. It is at intermediate values of θ where the ratio threshold technique may be an improvement over linear combining.

We now derive the bit error probability p_b for the ratio threshold test with linear combining. There are two different ways a diversity reception with interference can be accepted. Suppose that symbol 0 is sent, and let

$$p_z = P^I(R_{0,l} > R_{1,l}\theta) \triangleq P(R_{0,l} > R_{1,l}\theta \mid \text{interference + noise present})$$

and

$$p_e = P^I(R_{1,l} > R_{0,l}\theta) \triangleq P(R_{1,l} > R_{0,l}\theta \mid \text{interference + noise present}).$$

The sum of p_c and p_e is the probability that a diversity reception with interference is accepted. Thus, $1 - p_c - p_e$ is the probability that a diversity reception with interference is rejected. The probabilities p_c and p_e may be written as [10]

$$p_c = 1 - \frac{\theta^2 \exp\{-\nu_I^2/(2(1+\theta^2))\}}{1+\theta^2} \tag{4.2}$$

and

$$p_e = \frac{\exp\{-\nu_I^2\theta^2/(2(1+\theta^2))\}}{1+\theta^2}. \tag{4.3}$$

where ν_I is defined in (3.4).

Similarly, there are two different ways a diversity reception with interference absent can be accepted. We define the quantities p_r and p_w for divers y receptions with interference absent by replacing ν_I by ν_N in (4.2) and (4.3), respectively. For $\theta=1$, p_r and p_w are the probability of correct reception and the probability of error for a system with binary orthogonal signaling and no diversity in additive white Gaussian noise. The probability that $R_{0,l} > R_{1,l}\theta$ for a given diversity reception is

$$P_C = \rho p_c + (1-\rho)p_r, \tag{4.4}$$

and the probability that $R_{1,l} > R_{0,l}\theta$ for a given diversity reception is

$$P_E = \rho p_e + (1-\rho)p_w. \tag{4.5}$$

Thus, the probability that a diversity reception is rejected is $1-P_C-P_E$.

The probability of error for a system using the ratio threshold technique with linear diversity combining can be written as

$$p_b = \sum_{n=0}^{L} \sum_{m=0}^{L-n} C(L; m, n, L-n-m) P_E^m P_C^n (1-P_E-P_C)^{L-m-n} \; P(e; m, n), \tag{4.6}$$

where $C(N; N_1, N_2, ..., N_r)$ is the multinomial coefficient defined as $N!/(N_1!N_2! \cdots N_r!)$ for $\sum_{i=1}^{r} N_i = N$. The term $P(e; m, n)$ is the probability of error given that $n+m$ diversity receptions are accepted; n of the accepted diversity receptions have $R_{0,l} > R_{1,l}\theta$ and m of the accepted diversity receptions have $R_{1,l} > R_{0,l}\theta$. The number $L-m-n$ is the number of diversity receptions that are rejected. If $m=n=0$, then all the diversity receptions are linearly combined. Otherwise, only the accepted diversity receptions are linearly combined.

First consider $P(e; m, n)$ when $m \neq 0$ or $n \neq 0$. To compute this conditional probability of error, let Z denote the difference $R_{0,l} - R_{1,l}$. With probability ρ, interference is present on a diversity reception, and with probability $1-\rho$, interference is absent on a diversity reception. Thus, we may derive the density for Z by using a ρ-mixture model for the interference. That is, the density for Z may be written as ρ times the conditional density for Z given that the interference is present, plus $1-\rho$ times the conditional density for Z given that the interference is absent. We consider the two ways a diversity reception can be accepted, and find the conditional density for Z given each of these. Thus, we compute the conditional density of Z given that $R_{0,l} > R_{1,l}\theta$, denoted by $f_Z(z \mid R_{0,l} > R_{1,l}\theta)$, and the conditional density of Z given that $R_{1,l} > R_{0,l}\theta$, denoted by $f_Z(z \mid R_{1,l} > R_{0,l}\theta)$.

First consider the situation in which interference is present. Suppose $f_{0,1}^I(x, y)$ is the conditional joint density of $R_{0,l}$ and $R_{1,l}$ given that interference is present, so that $f_{0,1}^I(x, y \mid R_{0,l} > R_{1,l}\theta)$ is the conditional joint density of $R_{0,l}$ and $R_{1,l}$ given that $R_{0,l} > R_{1,l}\theta$ and that interference is present. Similarly, suppose $f_{0,1}^N(x, y)$ and $f_{0,1}^N(x, y \mid R_{0,l} > R_{1,l}\theta)$ are the corresponding conditional densities given that interference is absent. Then, for the ρ-mixture model for the interference, we may write

$$f_{0,1}(x, y \mid R_{0,l} > R_{1,l}\theta) = \rho f_{0,1}^I(x, y \mid R_{0,l} > R_{1,l}\theta) + (1-\rho) f_{0,1}^N(x, y \mid R_{0,l} > R_{1,l}\theta), \tag{4.7}$$

and

$$f_Z(z \mid R_{0,l} > R_{1,l}\theta) = \int_{-\infty}^{\infty} f_{0,1}(u+z, u \mid R_{0,l} > R_{1,l}\theta)du. \tag{4.8}$$

Using a form of Bayes' rule [21], note that

$$f^{I}_{0,1}(x, y \mid R_{0,l} > R_{1,l}\theta) = \frac{P^{I}(R_{0,l} > R_{1,l}\theta \mid R_{0,l}=x, R_{1,l}=y) f^{I}_{0,1}(x, y)}{p_c}.$$

The probability $P^I(R_{0,l} > R_{1,l}\theta \mid R_{0,l}=x, R_{1,l}=y)$ is trivial: it is unity over the region $x > y\theta$ and is 0 otherwise. Also, because the signals are orthogonal, the density $f^{I}_{0,1}(x, y)$ is the product of $f^{I}_{0}(x)$ and $f^{I}_{1}(y)$, the marginal densities of the envelope detector outputs given that the interference is present, defined in (3.6) and (3.7). After using Bayes' rule on the density $f^{N}_{0,1}(x, y \mid R_{0,l} > R_{1,l}\theta)$, we can write (4.8) as

$$f_Z(z \mid R_{0,l} > R_{1,l}\theta) = \int_0^{z/(\theta-1)} \left[\frac{\rho}{p_c} f^{I}_{0}(u+z) f^{I}_{1}(u) + \frac{(1-\rho)}{p_r} f^{N}_{0}(u+z) f^{N}_{1}(u) \right] du, \quad z > 0. \tag{4.9}$$

where the limits on the integration were found by examining the region in which $f_{0,1}(x, y \mid R_{0,l} > R_{1,l}\theta)$ is nonzero. A similar derivation shows that the density of Z conditioned on $R_{1,l} > R_{0,l}\theta$ is

$$f_Z(z \mid R_{1,l} > R_{0,l}\theta) = \int_0^{z/(1-\theta)} \left[\frac{\rho}{p_e} f^{I}_{0}(u) f^{I}_{1}(u-z) + \frac{(1-\rho)}{p_w} f^{N}_{0}(u) f^{N}_{1}(u-z) \right] du, \quad z < 0. \tag{4.10}$$

Once we have computed the densities in (4.9) and (4.10), the density function for the sum of the Z for accepted diversity receptions, denoted by $f(\zeta; m, n)$, is the convolution of the $(n-1)$-fold self-convolution of $f_Z(z \mid R_{0,l} > R_{1,l}\theta)$ with the $(m-1)$-fold self-convolution of $f_Z(z \mid R_{1,l} > R_{0,l}\theta)$. (Recall that an r-fold self-convolution of f is f if $r=0$, it is $f*f$ if $r=1$, etc. It is $\delta(\cdot)$ if $r=-1$.) The conditional probability of error $P(e; m, n)$ is the integral of $f(\zeta; m, n)$ over all negative values of ζ, provided that $m \neq 0$ or $n \neq 0$.

The conditional probability of error $P(e; 0, 0)$ in (4.6) is the probability of error given that all of the diversity receptions are rejected by the ratio threshold test. We need to find the conditional density $f_Z(z \mid \theta^{-1} < R_{0,l}/R_{1,l} < \theta)$. By using Bayes' rule as before and after finding the appropriate limits of integration, we can write the conditional density for Z as

$$f_Z(z \mid \theta^{-1} < R_{0,l}/R_{1,l} < \theta)$$

$$= \begin{cases} \displaystyle\int_{z/(\theta-1)}^{\infty} \left[\frac{\rho}{1-p_c-p_e} f_0^I(u+z) f_1^I(u) + \frac{1-\rho}{1-p_r-p_w} f_0^N(u+z) f_1^N(u) \right] du, & z \geq 0 \\ \displaystyle\int_{z/(1-\theta)}^{\infty} \left[\frac{\rho}{1-p_c-p_e} f_0^I(u) f_1^I(u-z) + \frac{1-\rho}{1-p_r-p_w} f_0^N(u) f_1^N(u-z) \right] du, & z < 0. \end{cases} \tag{4.11}$$

The probability $P(e:0,0)$ is found by taking the $(L-1)$-fold self-convolution of the conditional density for Z in (4.11), and then integrating the resulting density from $-\infty$ to 0.

In the numerical calculation of p_b for the ratio threshold test and linear combining, we can normalize the density functions in (4.9)-(4.11) with respect to σ_N so that p_b is a function of ρ, L, θ, E_b/N_0, and E_b/N_I. We do not need to specify the signal output voltage β or the noise densities N_0 and N_I explicitly. Due to the normalization, the ratio σ_N/σ_I is involved in the calculation, but this ratio is just ν_I/ν_N, where ν_I and ν_N are defined in (3.4) and (3.5).

4.2 Majority Logic Decoding

Another diversity combining scheme based on the ratio threshold test uses majority logic decoding for a system with binary orthogonal signaling. A hard decision is made on each accepted diversity reception. The probability that a correct decision is made on an accepted diversity reception is P_C, defined in (4.4), and the probability that the decision is incorrect is P_E, defined in (4.5). The probability that a diversity reception is rejected is $1-P_E-P_C$. After all L diversity receptions corresponding to the bit are tested, majority logic decoding is used on the accepted diversity receptions. In this scheme, it is assumed that in the case of ties and in the case in which all of the diversity receptions are rejected, the probability of a correct decision is $\frac{1}{2}$. Then, the average bit error probability for the ratio threshold test and majority logic is [10]

$$p_b = \frac{(1-P_E-P_C)^L}{2} + \sum_{i=1}^{L} \binom{L}{i} (1-P_E-P_C)^{L-i}$$

$$\cdot \left\{ \sum_{m=\lceil \frac{i+1}{2} \rceil}^{i} \binom{i}{m} P_E^m P_C^{i-m} + \frac{1}{2} \left(\left\lceil \frac{i+1}{2} \right\rceil - \left\lfloor \frac{i+1}{2} \right\rfloor \right) \binom{i}{i/2} (P_E P_C)^{i/2} \right\}, \tag{4.12}$$

where $\lceil x \rceil$ is defined as the smallest integer greater than or equal to x, and $\lfloor x \rfloor$ is defined as the largest integer less than or equal to x. Notice that ties can occur if L is odd, except that ties do not occur if an odd number of diversity receptions is accepted.

To calculate ρ^* for this scheme, consider interference with arbitrary power and distribution. The ratio threshold test does not mitigate the worst type of interference. In the absence of quiescent noise, an error can occur on any diversity reception with interference present, but an error does not occur on a diversity reception with interference absent. Thus, if interference is present on $\lceil \frac{L+1}{2} \rceil$ or more diversity receptions for a given bit, an error is made. Given the desired bit error probability p_b, ρ^* is the solution for ρ in the equation

$$p_b = \sum_{j=\lceil \frac{L+1}{2} \rceil}^{L} \binom{L}{j} \rho^j (1-\rho)^{L-j} . \tag{4.13}$$

A comparison of (4.13) and (4.1) shows that the value of ρ^* for majority logic decoding is better than ρ^* for linear combining for $L \geqslant 3$.

In Figure 4.1, we demonstrate the sensitivity of the bit error probability to θ at small ρ for the ratio threshold test with linear combining and for the ratio threshold test with majority logic decoding. It is at these small values of the interference duty factor where the ratio threshold technique is useful. Consider the curves for the ratio threshold test with linear combining (shown as solid curves in Figure 4.1). Recall that for $\theta=1$, the ratio threshold test with linear combining is equivalent to standard linear combining. In the example shown, there is a value

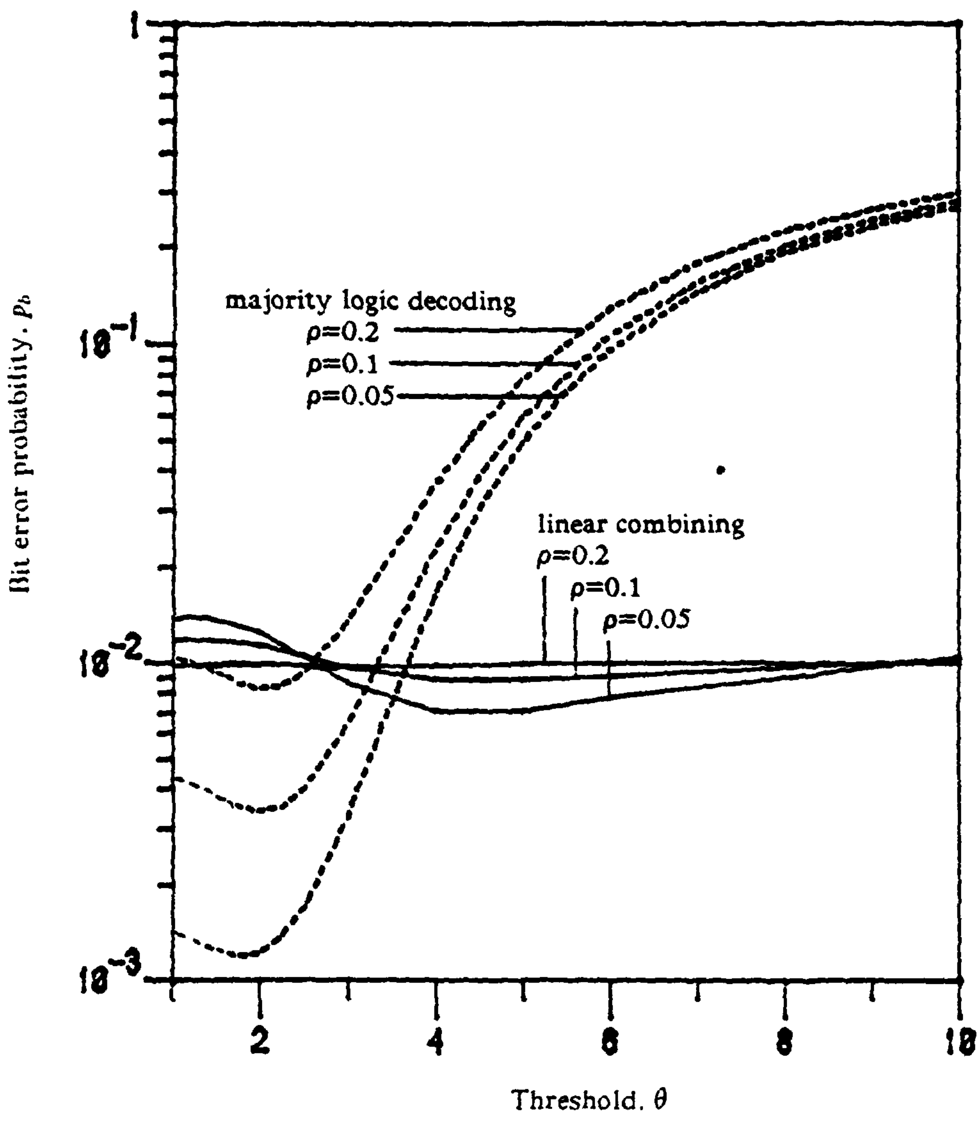

Figure 4.1. Probability of error p_b versus the threshold θ for small interference duty factors. $L = 3$, $E_b/N_0 =$ 18.0dB, and $E_b/N_I =$ 12.0dB for the ratio threshold test with diversity combining of θ greater than 1 that gives the minimum probability of error for each $\rho<0.2$. Thus, for small interference duty factors, the ratio threshold technique can do better than linear combining. A threshold of around $\theta=4$ works well for this scheme. For large values of θ, the combining scheme approximates linear combining because nearly all diversity receptions are rejected and included in the combining. Thus, the bit error probability p_b is the same for $\theta=1$ and $\theta=\infty$.

Now consider the curves in Figure 4.1 that correspond to the ratio threshold test and majority logic decoding (shown as dashed curves). The probability of error is much more sensitive to θ for majority logic decoding than for linear combining. The minimum error probability for majority logic decoding is smaller than the minimum for linear combining, which indicates that majority logic decoding can perform better than linear combining against narrowband interference. A threshold of $\theta=2$ works well for the ratio threshold test and majority logic decoding. The probability of error goes to $\frac{1}{2}$ as $\theta\rightarrow\infty$ because nearly all diversity receptions are rejected and random decisions are made.

The performance of the ratio threshold test with linear combining is shown in Figure 4.2 for $L=1$, 2, and 3. The curve for $L=1$ is independent of θ. In terms of narrowband interference rejection capability, $L=2$ is better than $L=1$. However, $L=3$ does not improve narrowband interference rejection over $L=2$.

The performance of the ratio threshold test with majority logic decoding is shown in Figure 4.3 for $L=1$, 2, and 3 with $\theta=1$, and for $L=2$ and $L=3$ with $\theta=2$. Narrowband interference rejection is improved as the diversity level is increased. For $L\geqslant 3$, the ratio threshold test with majority logic decoding has larger values of ρ_{min} than the ratio threshold test with linear combining. However, majority logic decoding does not perform as well as linear combining against interference which has a large duty factor.

The intuitive reason for the superiority of majority logic decoding for narrowband interference rejection is illustrated by considering the example with diversity level 3. Consider

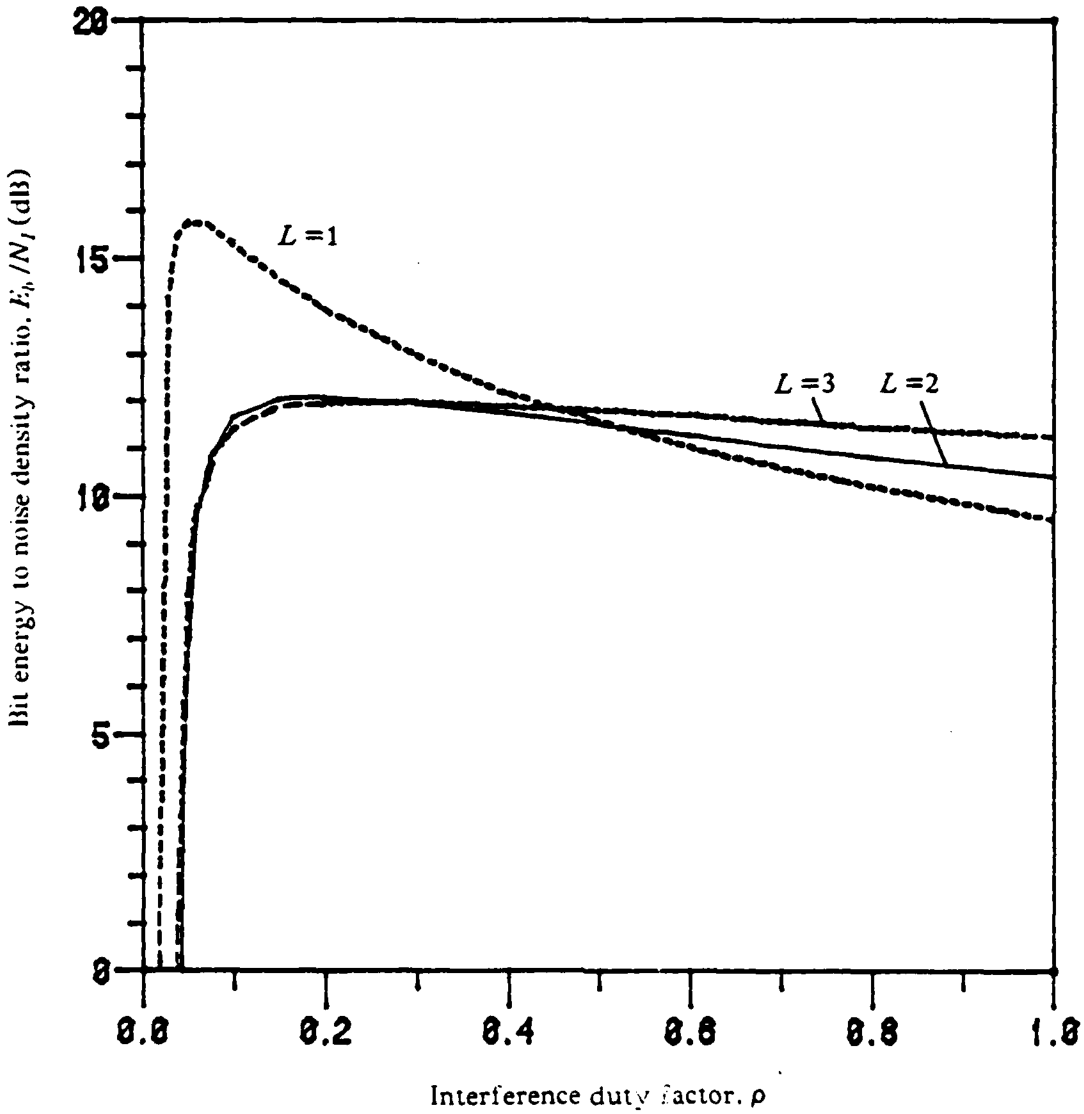

Figure 4.2. Bit energy to noise density ratio versus interference duty factor for $M = 2$, $p_b = 0.01$, $E_b/N_0 = 18$dB, and diversity levels 1 through 3 for the ratio threshold test with $\theta = 4$ and linear combining

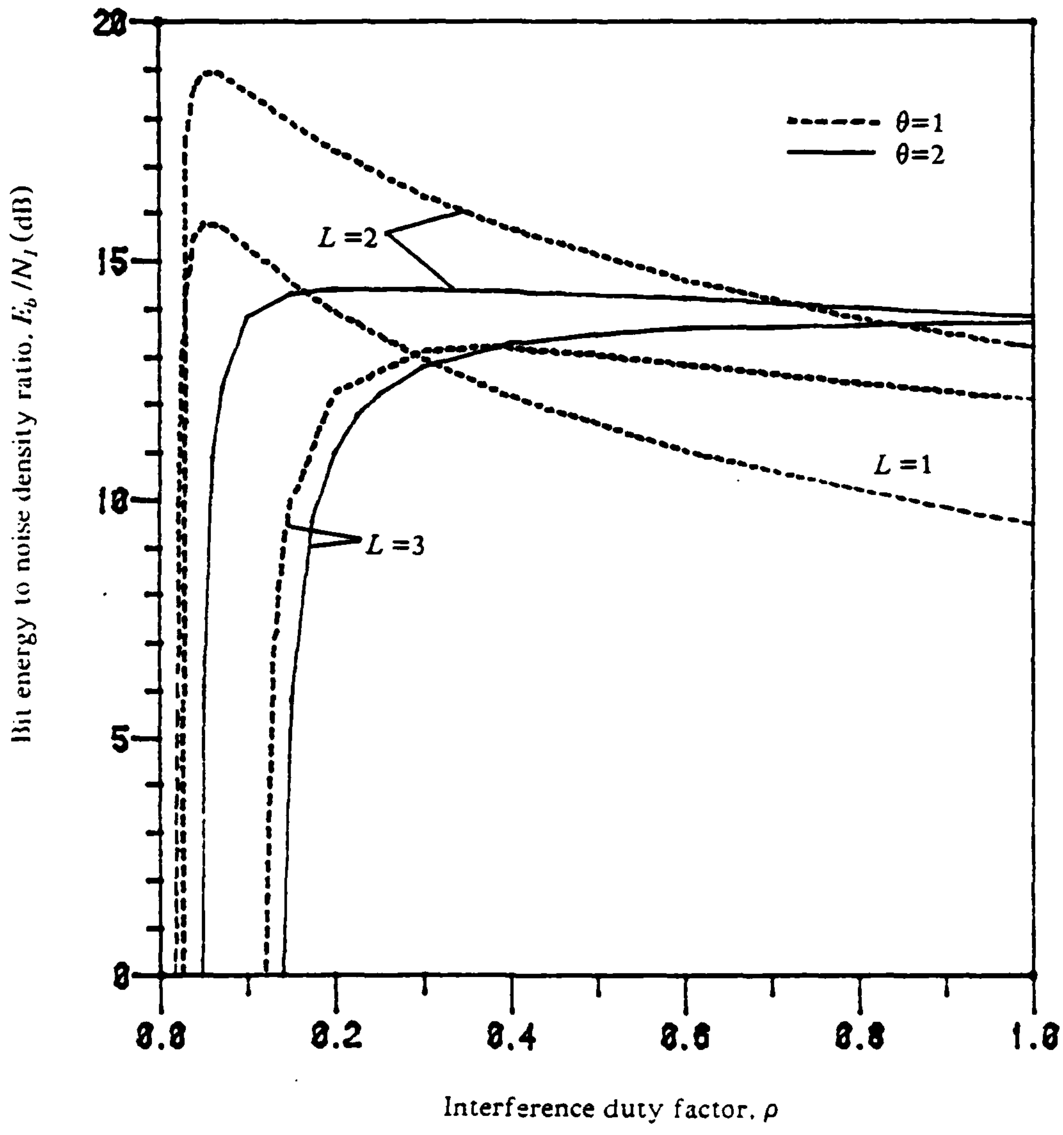

Figure 4.3. Bit energy to noise density ratio versus interference duty factor for $M = 2$, $p_h = 0.01$, $E_b/N_0 = 18$dB, and diversity levels 1 through 3 for majority logic decoding, and diversity levels 2 and 3 for the ratio threshold test with $\theta=2$ and majority logic decoding the situation in which three diversity receptions are accepted. Suppose $R_{0,l} > R_{1,l}\theta$ for two diversity receptions, and $R_{1,l} > R_{0,l}\theta$ for one diversity reception. There cannot be an error for majority logic decoding in this situation. However, there is a possibility of an error for linear combining. A situation that favors linear combining is as follows. Suppose $R_{0,l} > R_{1,l}\theta$ for one diversity reception and $R_{1,l} > R_{0,l}\theta$ for the other two diversity receptions. A majority logic decoder makes an error with probability 1 in this situation, while a system that employs linear combining makes an error with probability less than 1. If ρ is small, the first situation has higher probability than the second situation. Also, other possible situations do not heavily favor either linear combining or majority logic decoding. For example, the situation in which two diversity receptions are accepted, one with $R_{0,l} > R_{1,l}\theta$ and the other with $R_{1,l} > R_{0,l}\theta$, does not heavily favor either diversity combining scheme. Furthermore, there cannot be an error for majority logic decoding or for linear combining for the situation in which all accepted diversity receptions have $R_{0,l} > R_{1,l}$. An error is made with probability 1 by both of the diversity combining schemes for the situation in which all accepted diversity receptions have $R_{1,l} > R_{0,l}$. Thus, we conclude that for $L=3$, majority logic decoding has an advantage over linear combining in the presence of narrowband interference.

As the diversity level increases above $L=3$, majority logic decoding increases its advantage over linear combining in narrowband interference, because there are more situations with high probability that favor majority logic decoding. For small ρ, the interference is strong when it is present on a diversity reception. Majority logic decoding does well in spite of the strong interference, because errors do not "propagate" in a majority logic decoding scheme. However, interference "propagates" within the sum of L diversity receptions in the linear combining scheme.

In the next section, we show that $\rho_{\min}$ can be increased for the ratio threshold test with linear combining. The value of E_b/N_I can be decreased for large values of ρ for the ratio threshold test with majority logic decoding. This improvement is gained by changing the strategy of the linear combining and the majority logic decoding schemes for situations in which all diversity receptions are rejected.

4.3 Variation on Linear Combining and Majority Logic Decoding

The strategy of combining all of the diversity receptions when all are rejected is not optimum. We may want to use the ratio T_l once again to determine which of the rejected diversity receptions to combine. In this section, we consider a modification of the linear combining and the majority logic decoding techniques used with the ratio threshold test. We discuss the scheme that uses only the diversity reception with the largest ratio statistic for the situation in which all of the diversity receptions of a given bit are rejected. This variation can be applied to systems with diversity levels greater than 1.

For the variation on linear combining, we need to recalculate $P(e;0,0)$ and incorporate it into (4.6). For the variation on majority logic decoding, the factor of $\frac{1}{2}$ in the first term of (4.12) is replaced by the newly calculated $P(e;0,0)$. To begin the analysis of the probability of error $P(e;0,0)$, we note a few useful facts. Let $\{R_{0,l}: 1 \leqslant l \leqslant L\}$ and $\{R_{1,l}: 1 \leqslant l \leqslant L\}$ denote the envelope detector outputs for the L diversity receptions of a given bit and let $\tau_l = \dfrac{R_{1,l}}{R_{0,l}}$ for each l. Given that all the diversity receptions are rejected, we have that $\theta^{-1} < \tau_l < \theta$ for all l. Also, for analytical purposes, an error occurs if the symbol 0 is sent and $\max\limits_l \tau_l > \max\limits_l \dfrac{1}{\tau_l}$. But, note that $\max\limits_l \dfrac{1}{\tau_l} = \dfrac{1}{\min\limits_l \tau_l}$. Finally, if the symbol 0 is sent and all diversity receptions are rejected, there can be no error for $\max\limits_l \tau_l < 1$ since this would imply $R_{0,l} > R_{1,l}$ for all l.

We define the random variables $U = \max\limits_l \tau_l$, $W = \min\limits_l \tau_l$, and $V = W^{-1}$ in order to simplify the expression for the probability of error. We find the ρ-mixture density and distribution functions of τ_l as follows. If the symbol 0 is sent, τ_l is the ratio of a Rayleigh distributed random variable to a Rician distributed random variable. Let $f_\tau^l(t)$ and $F_\tau^l(t)$ denote the conditional density and distribution functions for the τ_l given that the interference is present. Then

$$f_\tau^I(t) = \int_0^\infty tx^3 \exp\{-\frac{x^2(t^2+1)+v_I^2}{2}\} I_0(v_I x)dx\,,\ t>0,$$

$$= \frac{t(2t^2+2+v_I^2)}{(t^2+1)^3}\exp\{-\frac{v_I^2t^2}{2(t^2+1)}\},\ t>0, \tag{4.14a}$$

and

$$F_\tau^I(t) = 1 - \int_0^\infty x \exp\{-\frac{x^2(t^2+1)+v_I^2}{2}\} I_0(v_I x)dx\,,\ t>0,$$

$$= 1 - \frac{1}{t^2+1}\exp\{-\frac{x^2(t^2+1)+v_I^2}{2}\} I_0(v_I x)dx\,,\ t>0. \tag{4.14b}$$

where $I_0(\cdot)$ is the 0-*th* order modified Bessel function. The density and distribution functions of τ_l for a diversity reception with interference absent, denoted by $f_\tau^N(t)$ and $F_\tau^N(t)$, are found by replacing v_I in (4.14) by v_N. We may write the ρ-mixture density and distribution functions for τ_l as

$$f_\tau(t) = \rho f_\tau^I(t) + (1-\rho) f_\tau^N(t),\ \ t>0, \tag{4.15a}$$

and

$$F_\tau(t) = \rho F_\tau^I(t) + (1-\rho) F_\tau^N(t),\ t>0. \tag{4.15b}$$

The bit error probability given that all the diversity receptions are rejected can be expressed as

$$P(e;0,0) = P(U>V \mid \theta^{-1}<W<U,\ 1<U<\theta)\cdot P(U>1 \mid \theta^{-1}<U<\theta)$$

$$= \frac{P(U^{-1}<V<U,\ 1<U<\theta)}{P(\theta^{-1}<W<U,\ 1<U<\theta)}\cdot\frac{P(1<U<\theta)}{P(\theta^{-1}<U<\theta)}. \tag{4.16}$$

The joint density function of U and V, the joint density function of U and W, and the distribution function of U, which can be written in terms of $f_\tau(t)$ and $F_\tau(t)$ in (4.15), are required to solve (4.16). The distribution function of U is just $[F_\tau(u)]^L$. Thus, we may write

$$P(U>1|\theta^{-1}<U<\theta)=\frac{[F_\tau(\theta)]^L-[F_\tau(1)]^L}{[F_\tau(\theta)]^L-[F_\tau(\theta^{-1})]^L}. \tag{4.17}$$

The joint density function of U and W is

$$f_{U,W}(u,w)=L(L-1)[F_\tau(u)-F_\tau(w)]^{L-2}f_\tau(u)f_\tau(w),\ 0<w<u<\infty, \tag{4.18}$$

and by making a typical transformation of variables, we can write the joint density function of U and $V=W^{-1}$ as

$$f_{U,V}(u,v)=L(L-1)v^{-2}[F_\tau(u)-F_\tau(v^{-1})]^{L-2}f_\tau(u)f_\tau(v^{-1}),\ 0<u^{-1}<v<\infty. \tag{4.19}$$

Using the joint densities given in (4.18) and (4.19), we have that

$$P(U>V\,|\,\theta^{-1}<W<U,\,1<U<\theta)=\frac{\int_1^\theta\int_{u^{-1}}^u f_{U,V}(u,v)\,dv\,du}{\int_1^\theta\int_{\theta^{-1}}^u f_{U,W}(u,w)\,dw\,du}. \tag{4.20}$$

The desired conditional probability of error is found by substituting (4.17) and (4.20) into (4.16).

Comparisons are made in Figures 4.4 and 4.5 among the diversity combining schemes which use the ratio threshold test. Clipped linear combining is also included in the comparison. In Figure 4.4, the diversity level is $L=2$. For small ρ, all of the combining techniques using the ratio threshold test perform better than standard linear combining (θ=1). The variation on linear combining with the ratio threshold test has the best value of ρ_{min}, and, for large values of ρ, its performance is about the same as that of standard linear combining.

Figure 4.5 gives a comparison of the diversity combining schemes for diversity level 3. Majority logic decoding is the better scheme, out of those using the ratio threshold test, to employ for narrowband interference rejection. The variation on majority logic gives more than 1dB improvement over majority logic decoding without the variation for large interference duty factors. The performance of the ratio threshold test with linear combining is disappointing. Although the variation on linear combining has a better value of ρ_{min} than linear combining without the variation, there is no improvement for diversity $L=3$ over $L=2$.

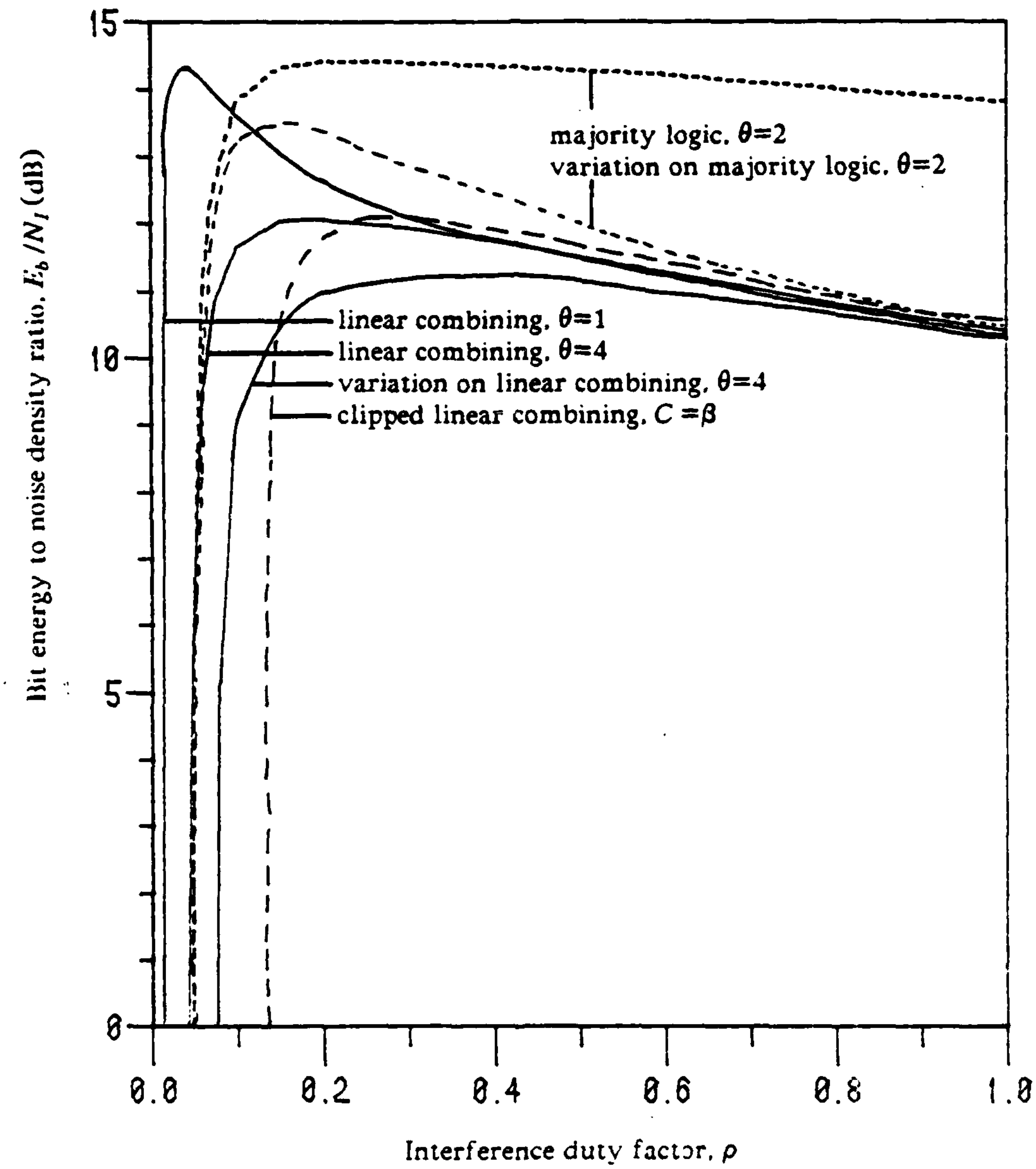

Figure 4.4. The ratio threshold test with diversity combining versus linear combining versus clipped linear combining for $L=2$, E_b/N_0=18dB, and $p_b=0.01$

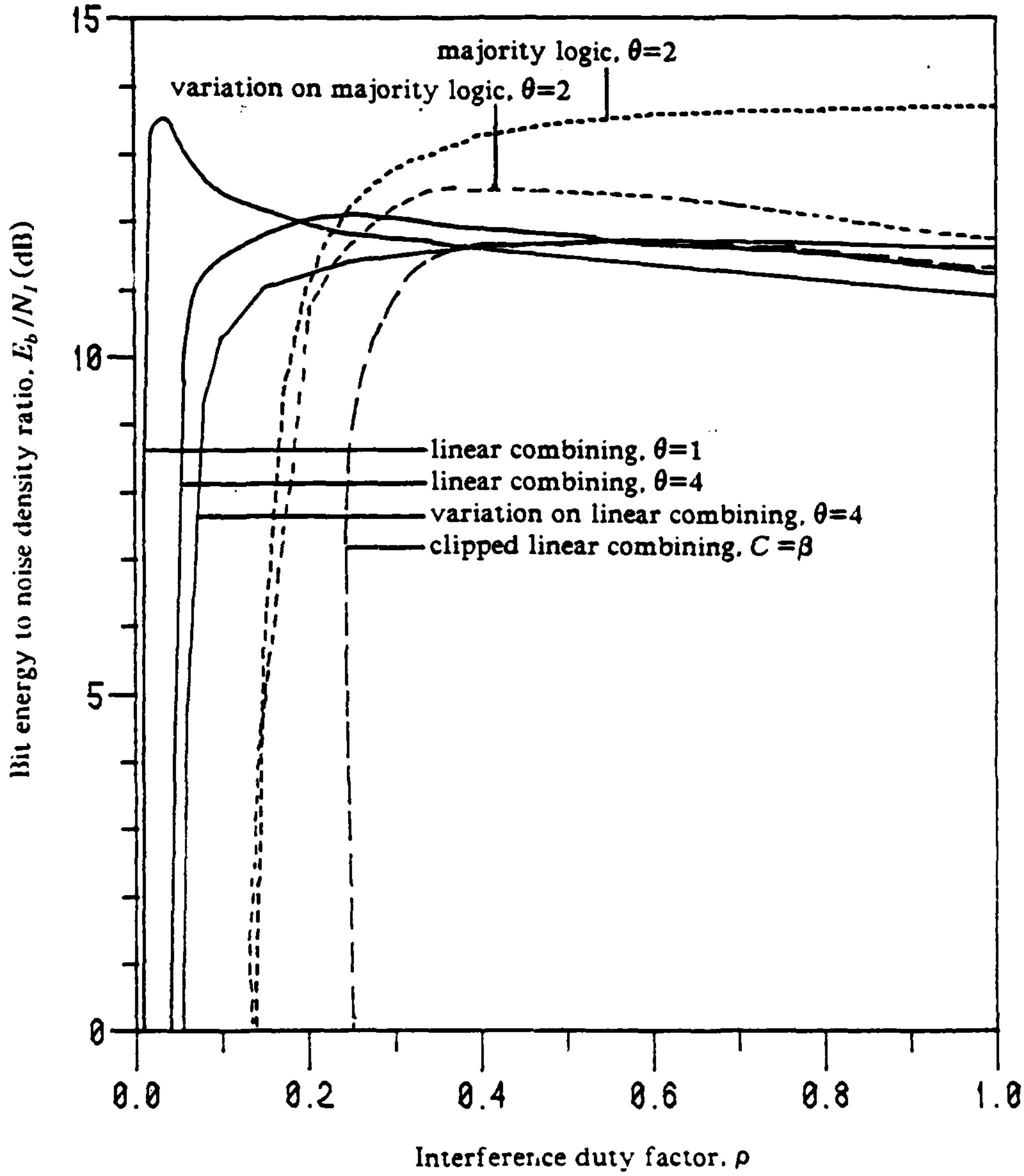

Figure 4.5. The ratio threshold test with diversity combining versus linear combining versus clipped linear combining for $L = 3$, $E_b/N_0 = 18$dB, and $p_b = 0.01$

Figures 4.4 and 4.5 illustrate that clipped linear combining is better than the ratio threshold technique in terms of narrowband interference rejection capability. However, the ratio threshold test with diversity is an improvement over standard linear combining. The ratio threshold test with majority logic decoding appears to be a good technique for systems with no side information. Although ρ_{min} for the ratio threshold technique is not as high as ρ_{min} for clipped linear combining, implementation considerations may favor the ratio threshold technique. That is, there are situations in which it may be difficult to set the clipping level for clipped linear combining, but in which the ratio threshold test would still work.

It should be noted that for the diversity scheme employing the ratio threshold test with the variation on majority logic decoding, the error probability is not as sensitive to the threshold θ as it is in Figure 4.1 for the scheme with the ratio threshold test with majority logic decoding without the variation. Figure 4.6 presents curves for the sensitivity of p_b to θ for the ratio threshold test with the variation on majority logic decoding. A value of θ within the range of 2 to 4 works well for this scheme. Larger values of θ work better for majority logic decoding with the variation than for majority logic decoding without the variation. This is because for the situation in which all of the diversity receptions are rejected, a more probable situation for larger values of θ, it is better to apply the variation than to make a random decision. Notice that the minimum error probability p_b over the range of θ is smaller for the curves of Figure 4.6 than for the curves of Figure 4.1.

4.4 The Ratio Threshold Test for M-ary Orthogonal Signaling

In this section, we analyze a diversity combining scheme that uses the ratio threshold test for a system with M-ary orthogonal signaling. The ratio statistic T_l for the l-th diversity reception is the ratio of the largest envelope detector output to the second largest envelope detector output. As before, if $T_l > \theta$, the l-th diversity reception is accepted, and, if $T_l < \theta$, the diversity reception is rejected. A hard decision is made on each accepted diversity reception. If there is at least one accepted diversity reception, the symbol that corresponds to the envelope

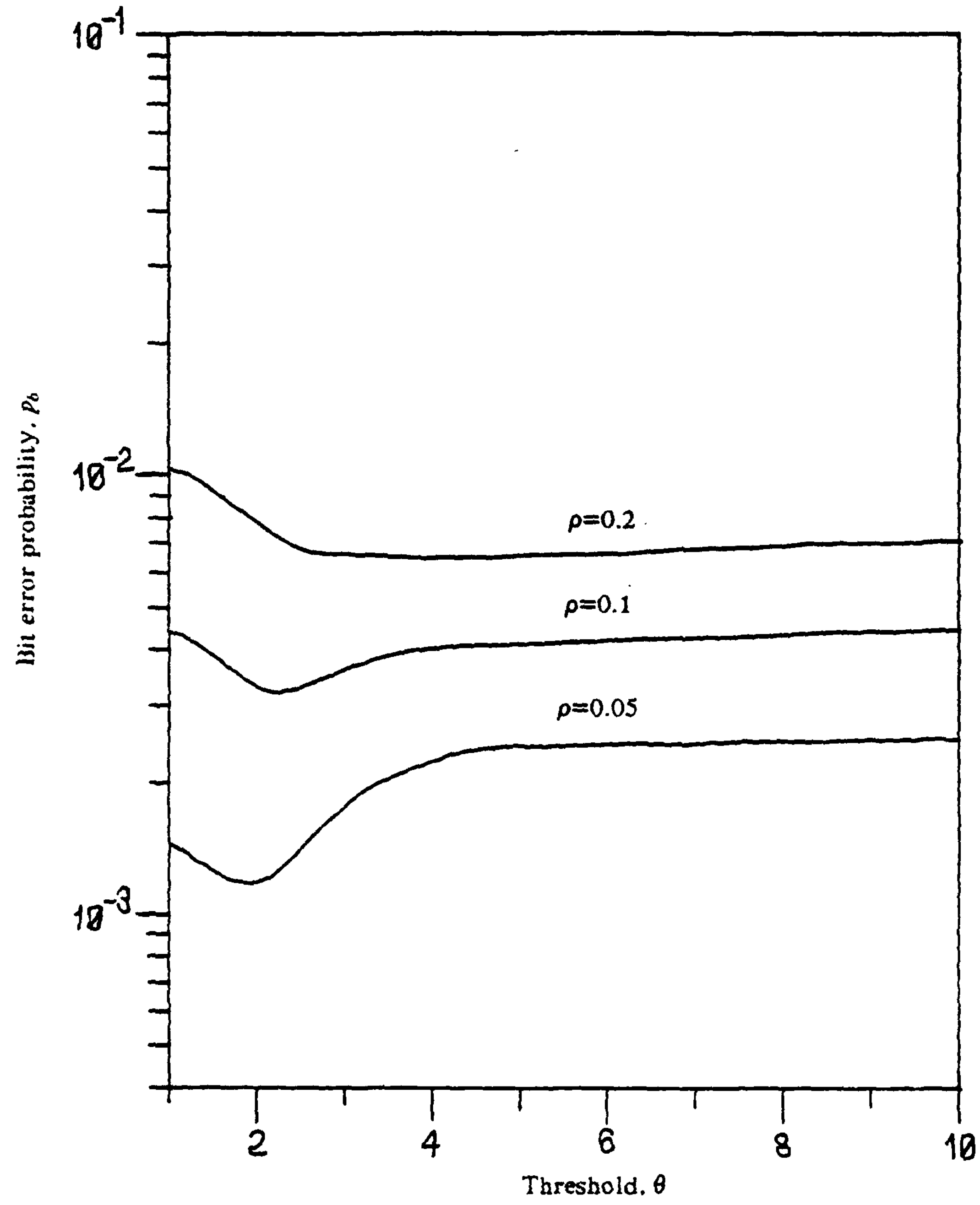

Figure 4.6. Probability of error p_b versus the threshold θ for small interference duty factors, $L=3$, E_b/N_0=18.0dB, and E_b/N_I=12.0dB for the ratio threshold test with the variation on majority logic decoding detector output with the maximum number of decisions in its favor is chosen. By a *tie*, we mean that two or more symbols have the same number of decisions in their favor, and all other symbols have a smaller number of decisions in their favor. In the case of a tie, a random decision is made among the symbols involved in the tie. If all of the diversity receptions are rejected, then the decision is based on the diversity reception with the largest ratio statistic.

Suppose the symbol 0 is sent. The envelope detector output $R_{0,l}$ for the l-th diversity reception is a Rician distributed random variable, and the other $M-1$ envelope detector outputs are Rayleigh distributed random variables. For each value of l, form the order statistics for the $M-1$ Rayleigh distributed envelope detector outputs as follows:

$$R'_{(1),l} \geqslant R'_{(2),l} \geqslant \cdots \geqslant R'_{(M-1),l}. \tag{4.21}$$

Three of the envelope detector outputs are of importance in determining the distribution of the ratio statistic T_l. $R'_{(1),l}$ will always be a component of the ratio statistic. The larger of the envelope detector outputs $R_{0,l}$ and $R'_{(2),l}$ will be the other envelope detector involved in the ratio statistic. The l-th diversity reception is accepted if $R'_{(1),l}/\max(R_{0,l}, R'_{(2),l})$ is either greater than θ or less than θ^{-1}. Otherwise, the l-th diversity reception is rejected.

We need to calculate the probability that a diversity reception is accepted or rejected for M-ary orthogon. signaling. Consider a diversity reception with interference present. There are two different ways this diversity reception can be accepted. Assuming the symbol 0 is sent, it is desirable that $R_{0,l} > R'_{(1),l}\theta$, and undesirable that $R'_{(1),l} > \theta\max(R_{0,l}, R'_{(2),l})$. Let

$$p_c = P^I(R_{0,l} > R'_{(1),l}\theta)$$

and

$$p_e = P^I(R'_{(1),l} > \theta\max(R_{0,l}, R'_{(2),l})).$$

Let $f_0^I(x)$ and $F_0^I(x)$ denote the conditional density and distribution functions for $R_{0,l}$ given that the interference is present. Let $f_k^I(x)$ and $F_k^I(x)$ denote the conditional density and distribution functions for each of the other envelope detector outputs $\{R_{k,l} : 1 \leqslant k \leqslant M-1, 1 \leqslant l \leqslant L\}$

given that the interference is present. Then, the the conditional distribution function for $R'_{(1),I}$ given that the interference is present is

$$F^I_{(1)}(x) = [F^I_k(x)]^{M-1}.$$

The conditional joint density function for $R'_{(1),I}$ and $R'_{(2),I}$ is [22]

$$f^I_{(2),(1)}(x,y) = (M-1)(M-2)[F^I_k(x)]^{M-3} f^I_k(x) f^I_k(y).$$

Thus, we can write p_c and p_e as

$$p_c = \int_0^\infty [F^I_k(x\theta^{-1})]^{M-1} f^I_0(x)dx, \tag{4.22}$$

and

$$\begin{aligned} p_e &= P^I(R'_{(1),I} > R_{0,I}\theta \bigcap R'_{(1),I} > R'_{(2),I}\theta) \\ &= P^I(R'_{(1),I} > R_{0,I}\theta)\, P^I(R'_{(1),I} > R'_{(2),I}\theta) \\ &= \left[1 - \int_0^\infty [F^I_k(x\theta)]^{M-1} f^I_0(x)dx\right]\left[(M-1)\int_0^\infty [F^I_k(x\theta^{-1})]^{M-2} f^I_k(x)dx\right]. \end{aligned} \tag{4.23}$$

The sum of p_c and p_e is the probability that a diversity reception with interference is accepted. Thus, $1 - p_c - p_e$ is the probability that a diversity reception with interference is rejected. Note that the integrals in (4.22) and (4.23) can be written as sums of terms of alternating signs. For example, (4.22) may be written as

$$p_c = \sum_{n=0}^{M-1} (-1)^n \frac{\theta^2}{n+\theta^2} \exp\{-\frac{v_I^2 n}{2(n+\theta^2)}\}.$$

However, we choose to keep the expressions in integral form to avoid numerical problems that arise in calculations of the sums for $M > 16$ due to alternating signs on numbers with large magnitudes [23].

There are two different ways a diversity reception with interference absent can be accepted. We define the quantities p_r and p_w for diversity receptions with interference absent by replacing $f^I_k(x)$, $F^I_k(x)$, and $f^I_0(x)$ by $f^N_k(x)$, $F^N_k(x)$, and $f^N_0(x)$, respectively, in (4.22) and (4.23). For θ=1, p_r and p_w are the probability of correct reception and the probability of error for M-ary orthogonal signaling in additive white Gaussian noise with no diversity. Using the ρ-mixture model for the interference, we can write the probability that $R_{0,l} > R'_{(1),l}\theta$ for a given diversity reception as

$$P_C = \rho p_c + (1-\rho)p_r . \tag{4.24}$$

and the probability that $R'_{(1),l} > \theta \max(R_{0,l}, R'_{(2),l})$ for a given diversity reception as

$$P_E = \rho p_e + (1-\rho)p_w . \tag{4.25}$$

Thus, the probability that a diversity reception is rejected is $1-P_C-P_E$.

We now derive the conditional probability of symbol error given that at least one of the L diversity receptions is accepted. Let $i>0$ denote the number of diversity receptions that are accepted. Let m denote the number of diversity receptions with $R'_{(1),l} > \theta\max(R_{0,l}, R'_{(2),l})$. Thus, $i-m$ is the number of diversity receptions with $R_{0,l} > R'_{(1),l}\theta$. A hard decision is made on each of the accepted diversity receptions. Then the envelope detector output with the maximum number of decisions in its favor is chosen. An error occurs if more than $i-m$ of the m diversity receptions with $R'_{(1),l} > \theta\max(R_{0,l}, R'_{(2),l})$ favor the same symbol. That is, if say the k-th envelope detector output $R_{k,l} = R'_{(1),l}$ for $i-m+1$ or more accepted diversity receptions, then an error occurs. Note that if $i=m$, an error occurs with probability 1. Also, note that m must be greater than or equal to $i-m+1$, which implies $m \geq \lfloor \frac{i+1}{2} \rfloor$, for an error to be possible.

Let $p(e; M-1, m, i-m)$ denote the conditional probability that $i-m+1$ or more of the m diversity receptions with $R'_{(1),l} > \theta\max(R_{0,l}, R'_{(2),l})$ favor the same symbol. This probability is conditioned on the event that m diversity receptions have $R'_{(1),l} > \theta\max(R_{0,l}, R'_{(2),l})$, $i-m$ diversity receptions have $R_{0,l} > R'_{(1),l}$, and $i>1$. To find $p(e; M-1, m, i-m)$, consider the problem of distributing m indistinguishable balls randomly among $M-1$ distinct boxes. The probability $p(e; M-1, m, i-m)$ is the same as the probability that at least one of $M-1$ boxes holds at least $i-m+1$ balls. This analog to our problem applies because the random variables $R_{k,l}$ are identically distributed and statistically independent for each $k \neq 0$, and for each

l. The probability that $R_{k,l} = R'_{(1),l}$ for the l-th diversity reception is equal to $(M-1)^{-1}$ for each $k \neq 0$. In our model, a ball placed in the k-th box represents the situation in which the k-th envelope detector output is the maximum.

There are $\nu(M-1, m)$ total ways to distribute m balls in $M-1$ boxes, where

$$\nu(q, r) \triangleq \binom{q+r-1}{r}. \tag{4.26}$$

For our problem, we need to find what number of these combinations meet the condition that there is at least one box which holds $i-m+1$ or more balls. Alternatively, we can find the number of ways to distribute m balls among $M-1$ boxes so there are no more than $i-m$ balls in any given box, and subtract this result from the total number of combinations. This alternative view is a problem of unordered sampling with limited replacement. The solution to such a problem is the coefficient of t^m in the generating function $(1+t+\cdots+t^{i-m})^{M-1}$ [24]. The desired coefficient for our problem is $\eta(M-1, m, i-m)$, where

$$\eta(q, r, s) = \sum_{k=0}^{\lfloor \frac{r}{s+1} \rfloor} (-1)^k \binom{q}{k} \binom{r-ks-k+q-1}{r-ks-k}. \tag{4.27}$$

We define $\eta(0, 0, s)=1$ and $\binom{x}{y}=0$ for $y<0$. Note that if $r>qs$, (4.27) is equal to zero. That is, the number of balls must not be more than the number of boxes times the capacity of a box. Thus, if $m>(M-1)(i-m)$, then there are 0 combinations of balls in boxes that meet our requirement of no more than $i-m$ balls in any given box.

The conditional symbol error probability given that m diversity receptions have $R'_{(1),l} \geq \theta \max(R_{0,l}, R'_{(2),l})$, $i-m$ diversity receptions have $R_{0,l} > R'_{(1),l}\theta$, and $i>0$ (excluding ties involving the symbol 0) may be written as

$$p(e; M-1, m, i-m) = \begin{cases} 1; & m > i\dfrac{M-1}{M} \\ 1 - \dfrac{\eta(M-1, m, i-m)}{\nu(M-1, m)}; & m \leq i\dfrac{M-1}{M}. \end{cases} \tag{4.28}$$

The symbol error probability for $i > 0$ (excluding ties involving the symbol 0) is

$$P_{s,E} = \sum_{i=1}^{L} \binom{L}{i} (1 - P_E - P_C)^{L-i} \left[\sum_{m=\lfloor \frac{i+1}{2} \rfloor}^{\lfloor i \frac{M-1}{M} \rfloor} \binom{i}{m} P_E^m P_C^{i-m} p(e; M-1, m, i-m) \right.$$

$$\left. + \sum_{m=\lfloor i \frac{M-1}{M} \rfloor + 1}^{i} \binom{i}{m} P_E^m P_C^{i-m} \right]. \tag{4.29}$$

Next we calculate the symbol error probability due to ties for $i > 0$. Recall that in a tie, two or more symbols have the same number of decisions in their favor, and all other symbols have a smaller number of decisions in their favor. We are only concerned with ties involving the symbol 0. Ties that occur among the other $M-1$ symbols result in a symbol error, and this situation is included in the expression in (4.29).

A *two-way tie* is a tie between the symbol 0 and any one of the other $M-1$ symbols. Returning to the model of the balls in the boxes, a two-way tie occurs if exactly one of the $M-1$ boxes contains exactly $i-m$ balls, and the remaining $M-2$ boxes each have less than $i-m$ balls. Assume that the k-th box holds $i-m$ balls. Then there are $2m-i$ balls left to distribute among the remaining $M-2$ boxes. For there to be at least one combination possible, m must meet the condition $\lfloor \frac{i}{2} \rfloor \leq m \leq \lfloor \frac{i(M-1)-(M-2)}{M} \rfloor$. The lower limit on m is due to the fact that $2m-i \geq 0$ for a two-way tie to be possible. The upper limit on m is derived from the condition that $r \leq qs$ for $\eta(q, r, s) > 0$. Another requirement for a two-way tie is that i, the number of accepted diversity receptions, must be greater than 1. We assume that a random guess is made if a tie occurs. Thus, the probability of error given that there is a two-way tie is $\frac{1}{2}$. The probability of a two-way tie times the probability of error given that there is a two-way tie is

$$\frac{1}{2}(M-1)\sum_{m=\lfloor\frac{i}{2}\rfloor}^{\lfloor\frac{i(M-1)-(M-2)}{M}\rfloor}\frac{\eta(M-2,2m-i,i-m-1)}{\nu(M-1,m)}.$$

A *three-way tie* is a tie among the symbol 0 and any two of the other $M-1$ symbols. For a three-way tie to occur, M and i must be greater than or equal to 3. The probability of a three-way tie times the probability of error given that a three-way tie occurs is

$$\frac{2}{3}\binom{M-1}{2}\sum_{m=\lfloor\frac{2}{3}i\rfloor}^{\lfloor\frac{i(M-1)-(M-3)}{M}\rfloor}\frac{\eta(M-3,3m-2i,i-m-1)}{\nu(M-1,m)}.$$

By continuing on with this pattern, we find that the symbol error probability due to ties can be written as

$$p_{s,T}=\sum_{i=2}^{L}\binom{L}{i}(1-P_E-P_C)^{L-i}\sum_{j=1}^{\min(i-1,M-1)}\frac{j}{j+1}\binom{M-1}{j}$$

$$\cdot\sum_{m=\lfloor\frac{j}{j+1}i\rfloor}^{\lfloor\frac{i(M-1)-(M-1-j)}{M}\rfloor}\binom{i}{m}\frac{\eta(M-1-j,m-j(i-m),i-m-1)}{\nu(M-1,m)}P_E^mP_C^{i-m}. \tag{4.30}$$

Finally, we consider the case in which $i=0$. When all of the diversity receptions are rejected, a random decision is made. Thus, the symbol error probability given that all of the diversity receptions are rejected times the probability that all of the diversity receptions are rejected is

$$p_{s,R}=(1-P_E-P_C)^L\frac{M-1}{M}. \tag{4.31}$$

Then, the symbol error probability for the system with the ratio threshold test for M-ary orthogonal signaling is given by

$$p_s=p_{s,E}+p_{s,T}+p_{s,R}. \tag{4.32}$$

The variation discussed in Section 4.3 may be applied in this scheme. That is, for the situation in which all diversity receptions are rejected, the decision may be based on the diversity reception with the largest ratio. The derivation for the probability of error for this situation is set up in Section 4.3. We only need to calculate the density and distribution functions $f_\tau(t)$ and $F_\tau(t)$ for M-ary orthogonal signaling, where τ_l is now defined as

$$\tau_l = \frac{R'_{(1),l}}{\max(R_{0,l}, R'_{(2),l})}$$

for each $1 \leqslant l \leqslant L$. First, consider the conditional distribution of τ_l given that the interference is present. The conditional density and distribution functions for τ_l, given that the interference is present, may be written as

$$\begin{aligned} f_\tau^I(t) = {} & f_\tau^I(t \mid R_{0,l} > R'_{(1),l}) P^I(R_{0,l} > R'_{(1),l}) \\ & + f_\tau^I(t \mid R'_{(2),l} < R_{0,l} < R'_{(1),l}) P^I(R'_{(2),l} < R_{0,l} < R'_{(1),l}) \\ & + f_\tau^I(t \mid R_{0,l} < R'_{(2),l}) P^I(R_{0,l} < R'_{(2),l}) \end{aligned} \quad (4.33a)$$

and

$$\begin{aligned} F_\tau^I(t) = {} & F_\tau^I(t \mid R_{0,l} > R'_{(1),l}) P^I(R_{0,l} > R'_{(1),l}) \\ & + F_\tau^I(t \mid R'_{(2),l} < R_{0,l} < R'_{(1),l}) P^I(R'_{(2),l} < R_{0,l} < R'_{(1),l}) \\ & + F_\tau^I(t \mid R_{0,l} < R'_{(2),l}) P^I(R_{0,l} < R'_{(2),l}). \end{aligned} \quad (4.33b)$$

Each conditional density and distribution function given in (4.33) involves an integral with a complicated integrand, so that it is not trivial to compute (4.33) numerically. For example, we may write

$$\begin{aligned} & f_\tau^I(t \mid R_{0,l} < R'_{(2),l}) P^I(R_{0,l} < R'_{(2),l}) \\ & = (M-1)(M-2) \int_0^\infty x \, [F_k^I(x)]^{M-3} f_k^I(x) f_k^I(tx) \int_0^x f_0^I(y) \, dy dx, \quad t \geqslant 1. \end{aligned}$$

To compute $f_\tau^I(t)$ and $F_\tau^I(t)$, it is necessary to compute six such integrals. Also, similar computations must be done for $f_\tau^N(t)$ and $F_\tau^N(t)$. Then, the ρ-mixture density and distribution functions for τ_l should be formed as in (4.15). Finally, the resulting density and distribution functions for τ_l should be substituted into (4.16)-(4.20) to compute $p_{s,g}$ for the variation. Because of the lengthy computation required for the variation, our numerical results do not include examples of this diversity combining scheme. Instead, we use $p_{s,R}$ provided in (4.31).

Figure 4.7 illustrates the sensitivity of the symbol error probability p_s to the threshold θ for three values of M. The symbol error probability is very sensitive to the threshold. The scheme with the ratio threshold test for M-ary orthogonal signaling works well for small values of θ, and it works poorly for values of θ greater than 3. Care must be taken to choose a good value for the threshold. Figure 4.8 gives another analysis of the sensitivity to θ. The solid curves correspond to the performance of the ratio threshold test with 32-ary orthogonal signaling for $\theta=1.01$, $\theta=1.3$, and $\theta=1.7$. The signal to noise ratio requirement decreases with θ. However, $\rho_{\min}$ also decreases with θ.

The other curves included in Figure 4.8 are for clipped linear combining with $C=\beta$ (shown as the dashed curve), and for optimum combining for receivers with perfect side information (shown as the dotted curve). The value of $\rho_{\min}$ is greater for the ratio threshold test than it is for clipped linear combining in this example. But, this is at the expense of a high signal to noise ratio requirement. Recall, however, that the value of θ is chosen by the communications system designer, but the clipping level for clipped linear combining may vary due to the communications channel. We see from Figure 3.5 that the sensitivity of clipped linear combining to the clipping level has a similar behavior to the sensitivity of ratio threshold test to the threshold, but that this sensitivity is not in the hands of the system designer.

The curves in Figure 4.9 are for three values of M; namely, $M=2$, $M=8$, and $M=32$. The best value of $\rho_{\min}$ is for 32-ary orthogonal signaling. Both $M=8$ and $M=32$ are superior to $M=2$ for this example. However, there are examples in which $M=2$ is better than larger values of M. That is, there are values of θ that work better for the ratio threshold test with majority logic decoding than for M-ary orthogonal signaling. This fact is illustrated in Figure 4.7. However, the values of θ that work better for binary orthogonal signaling than for larger values of M, do not give good performance for any of the signaling schemes.

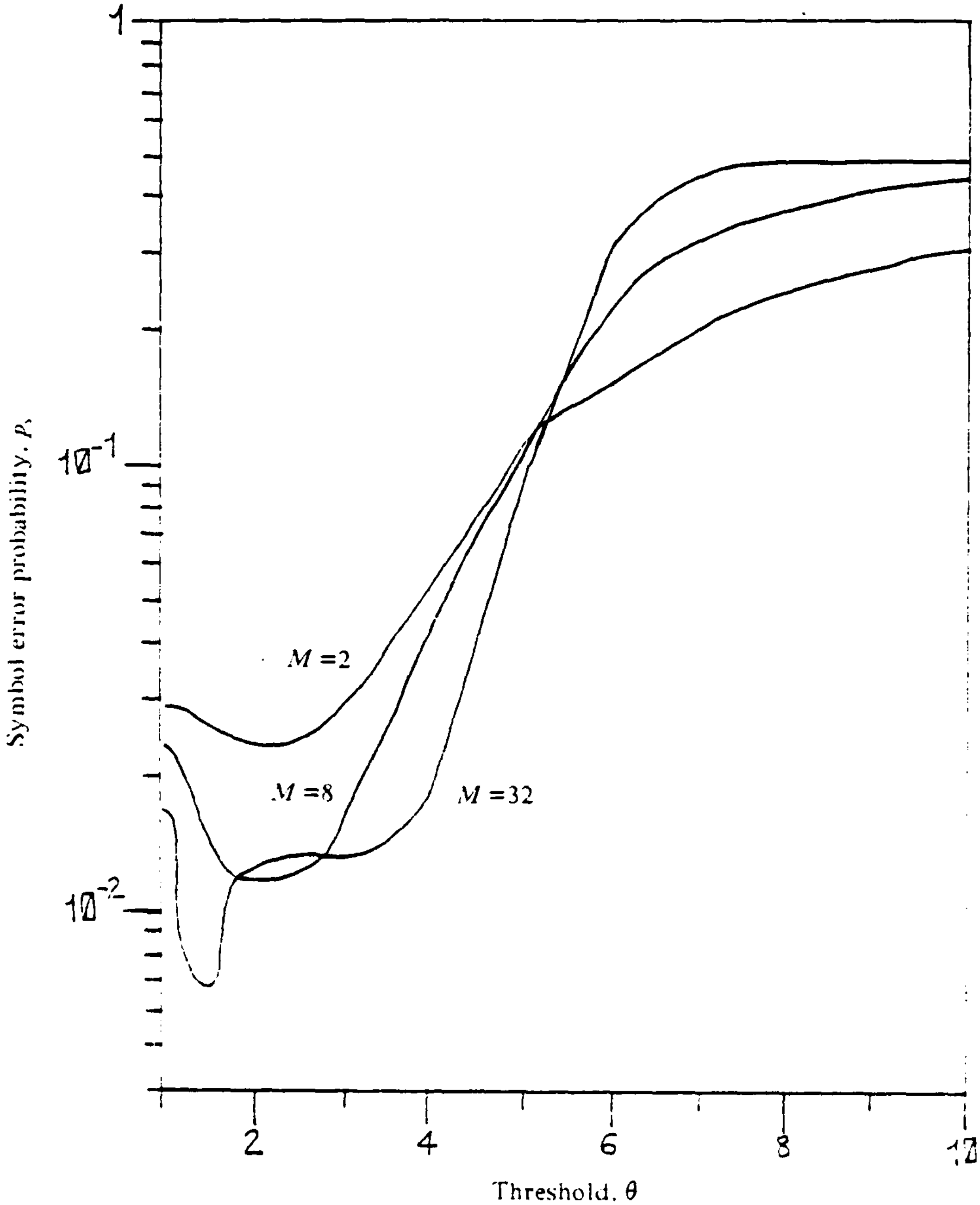

Figure 4.7. Probability of error p_b versus the threshold θ for L =3, E_b/N_0=18.0dB, E_b/N_I =9.0dB, ρ=0.3, and M =2, 8, 32, for the ratio threshold test with M-ary orthogonal signaling

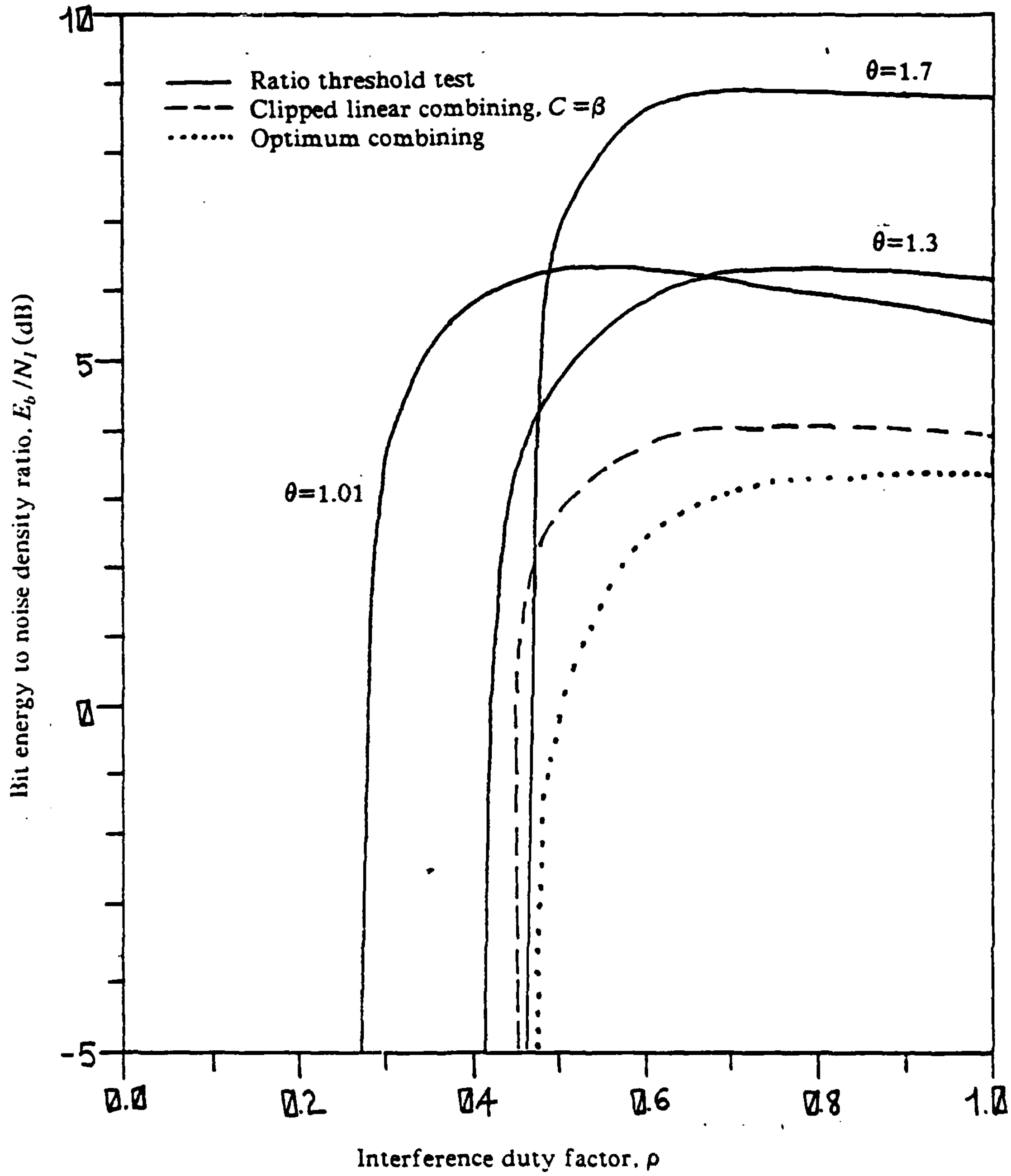

Figure 4.8. Bit energy to noise density ratio versus interference duty factor for $M=32$, $L=3$, $p_s=0.1$, and E_b/N_0=18dB for the ratio threshold test for M-ary orthogonal signaling with θ=1.01, 1.3, and 1.7, versus clipped linear combining with $C=\beta$, versus optimum combining

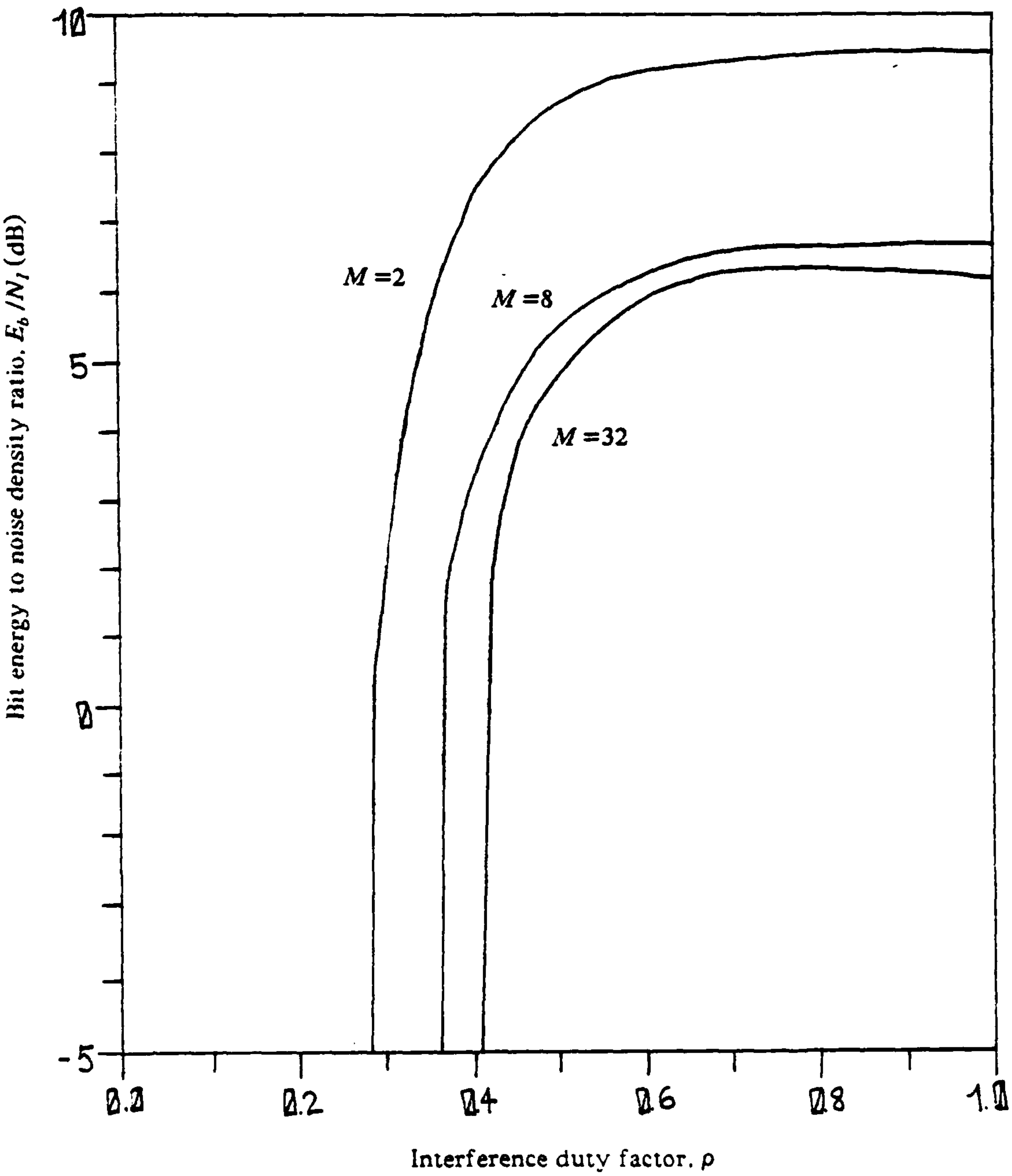

Figure 4.9. Bit energy to noise density ratio versus interference duty factor for $L=3$, $\theta=1.3$, $p_b=5.16\cdot10^{-2}$ ($p_s=0.1$ for $M=32$), and $E_b/N_0=18$dB for the ratio threshold test for M-ary orthogonal signaling

Figure 4.10 gives curves for the ratio threshold test with 32-ary orthogonal signaling, and curves for optimum combining, for diversity levels 3 and 5. The value of $\rho_{\min}$ for the ratio threshold test may be close to the value of ρ^* (and $\rho_{\min}$) for optimum combining, but the signal to noise ratio requirement for large values of ρ is as much as 5dB more for the suboptimal scheme. In spite of this larger requirement in signal to noise ratio, we conclude that the ratio threshold test for M-ary orthogonal signaling (including $M=2$ and the majority logic decoding scheme) is a good technique for systems without side information in the presence of partial-band interference. The narrowband interference rejection capability can be nearly as good as it is for systems with perfect side information.

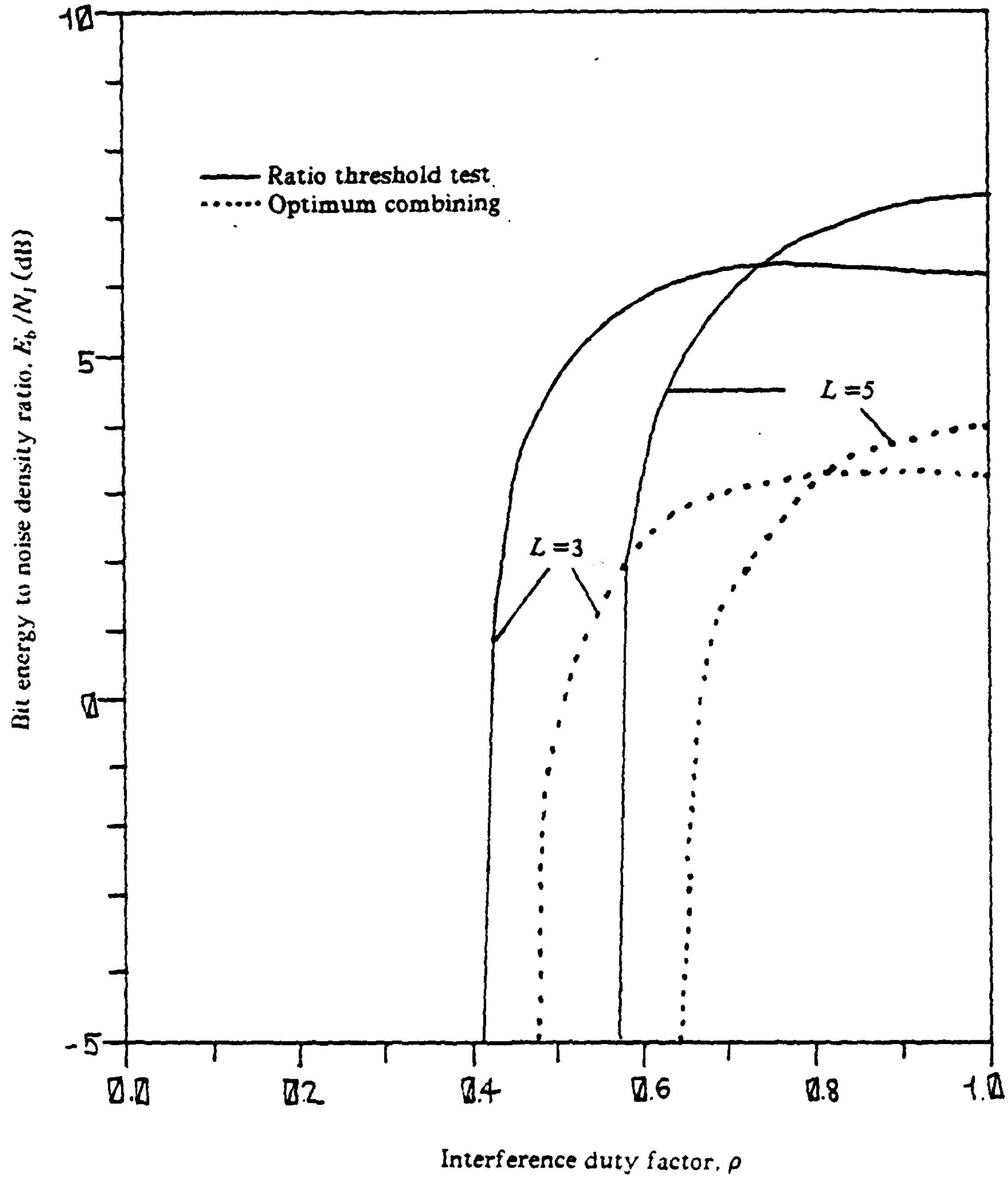

Figure 4.10. Bit energy to noise density ratio versus interference duty factor for θ=1.3, p_s=0.1, and E_b/N_0≈18dB for the ratio threshold test for 32-ary orthogonal signaling versus optimum *combining* for L=3 and L=5

CHAPTER 5

NONSELECTIVE FADING CHANNELS

In addition to partial-band interference, the communication channel may exhibit fading. In this chapter, we investigate the performance of the diversity combining schemes proposed in Chapter 4 for channels with partial-band interference and nonselective Rician fading.

5.1 Channel Model

In our model, we assume that the diversity receptions fade independently. In making this assumption for a system that uses frequency diversity, we are implying that the fading channel is characterized by frequency-selective fading. The *correlation bandwidth* of a frequency-selective fading channel is a parameter which describes how far apart two frequencies should be for the fading processes on these frequencies to be uncorrelated [25]. In an FH system, it is possible that the smallest spacing between adjacent frequency slots will not be be greater than the correlation bandwidth of the fading channel. But, it is not likely that the diversity transmissions for the same symbol will be sent over adjacent frequency slots. In fact, we may wish to design the hopping pattern to avoid this. Thus, by assuming that the diversity receptions fade independently, we are assuming that the smallest spacing between the frequency slots used by the diversity transmissions of the same symbol is larger than the correlation bandwidth of the channel.

We also assume that the fading is nonselective on each diversity reception. This assumption is valid as long as the bandwidth used by a given diversity transmission is much smaller than the correlation bandwidth of the fading channel. Thus, the wideband channel of the FH system is modeled as a group of independent narrowband channels, each with nonselective fading. We consider nonselective Rician fading.

For the channel with Rician fading and partial-band interference, we can describe each diversity reception as a signal with four components. Two components of the diversity reception are different versions of the diversity transmission: a non-faded version and a Rayleigh-faded version. The sum of these two components can be thought of as one Rician faded component. The fading attenuates the signal amplitude with a Rician distributed random variable [15]. The third component of the diversity reception is due to the Gaussian partial-band interference. Finally, the fourth component of the diversity reception is additive white Gaussian noise.

The density function for the amplitude of the received signal on a Rician fading channel is

$$f_A(a) = \frac{a}{\sigma^2}\exp\{-\frac{a^2+b^2}{2\sigma^2}\}I_0(\frac{ab}{\sigma^2}), \ a>0. \tag{5.1}$$

The quantity b^2 is the signal strength of the non-faded or *specular* component, and $2\sigma^2$ is the mean-squared value of the signal strength of the Rayleigh-faded or *scatter* component. Thus, the mean-squared signal strength of the sum of the two components is $b^2+2\sigma^2$. The ratio $\gamma^2 \triangleq 2\sigma^2/b^2$ is the ratio of the mean-squared value of the signal strength of the scatter component to the signal strength of the specular component. For $\gamma^2=\infty$, the channel model is a Rayleigh fading channel. For $\gamma^2=0$, there is no fading (only a constant attenuation equal to b) [26]-[28].

We use the performance measures described in Chapter 1 and used throughout Chapters 2-4. We define $\rho_{\min}$ to be the largest number such that a given error probability can be achieved in the presence of Gaussian quiescent noise, fading, and Gaussian partial-band interference with any duty factor less than this number. This parameter is useful in describing the system's narrowband interference rejection capability. We are also interested in the maximum signal to noise ratio required to achieve a desired error probability over the entire range of interference duty factors.

5.2 Clipped Linear Combining

As emphasized in Chapter 3, knowledge of the signal output voltage is required in order to effectively employ clipped linear combining. For a frequency-hop system, the signal output voltage may vary significantly from one frequency to the next because of different propagation characteristics of the communication channel at different frequencies. On a fading channel, the signal strength of any given diversity reception may vary significantly from the root-mean-squared signal strength. Thus, it may be difficult to establish the clipping level for frequency-hop communications over fading channels.

In a fading environment, effective communication makes use of the diversity receptions that have strong signal. However, a clipper tends to limit the diversity receptions that have strong signal in an attempt to limit the diversity receptions with partial-band interference. Thus, clipped linear combining is not useful for communications over fading channels.

The diversity combining schemes that use the ratio threshold test do not require the estimation of the signal strength. The threshold involved in the ratio threshold test is independent of the received signal output voltage, and the measure of the quality of a given diversity reception depends only on that diversity reception. Also, diversity receptions with strong signal are not likely to be rejected by the ratio threshold test. In the remainder of this chapter, we investigate the performance of the diversity combining schemes which use the ratio threshold test with diversity for FH systems subject to fading and partial-band interference.

5.3 The Ratio Threshold Test with Diversity Combining

First, we derive the probability of error for the system employing the ratio threshold test with the variation on linear combining. The analysis is for a system with binary orthogonal signaling. The probability of error has the same form as in Chapter 4, but most of the quantities must be modified to take into account the effects of fading. The densities $f_1^I(x)$ and $f_1^N(x)$, which do not depend on ν_I or ν_N, remain the same.

Let Γ_I^2 denote the average received signal to noise ratio given that interference is present. For a Rician fading channel, $\Gamma_I^2=(b^2+2\sigma^2)\frac{\nu_I^2}{2}$. Let $\beta_I^2=(b^2+2\sigma^2)\frac{E_b}{N_I}$ and $\beta_N^2=(b^2+2\sigma^2)\frac{E_b}{N_0}$. Then $\Gamma_I^2=\frac{1}{\beta_N^{-2}+\rho^{-1}\beta_N^{-2}}\frac{\log_2 M}{L}$. First, we find the quantities p_c and p_e. By replacing ν_I by $a\nu_I$

in (4.2) and (4.3), and then averaging a with respect to the density in (5.1), we obtain

$$p_c = 1 - \frac{\theta^2(\gamma^2+1)}{\Gamma_I^2\gamma^2+(1+\theta^2)(\gamma^2+1)}\exp\{-\frac{\Gamma_I^2}{\Gamma_I^2\gamma^2+(1+\theta^2)(\gamma^2+1)}\}. \tag{5.2}$$

and

$$p_e = \frac{\gamma^2+1}{\Gamma_I^2\gamma^2\theta^2+(1+\theta^2)(\gamma^2+1)}\exp\{-\frac{\Gamma_I^2\theta^2}{\Gamma_I^2\gamma^2\theta^2+(1+\theta^2)(\gamma^2+1)}\}. \tag{5.3}$$

Given that interference is absent, the average received signal to noise ratio is $\Gamma_N^2=\beta_N^2\frac{\log_2 M}{L}$. The probabilities p_r and p_w are found by replacing Γ_I by Γ_N in (5.2) and (5.3).

Next, we find the density function for $Z=R_{0,l}-R_{1,l}$, the difference between the envelope detector outputs. We need to find the conditional density for Z given that $R_{0,l}>R_{1,l}\theta$, the conditional density for Z given that $R_{1,l}>R_{0,l}\theta$, and the conditional density for Z given that $\theta^{-1}<R_{0,l}/R_{1,l}<\theta$. To find these conditional densities, we need to know the density for $R_{0,l}$ for a Rician fading channel. Suppose that interference is present on the diversity reception and that the symbol 0 is sent. Then the conditional density function for the envelope detector output corresponding to symbol 0 is

$$f_0^I(x) = x\exp\{-\frac{x^2\gamma^2+2}{2\gamma^2}\}\int_0^\infty \frac{a}{\sigma^2}\exp\{-\frac{a^2(v_I^2\sigma^2+1)}{2\sigma^2}\}I_0(xav_I)I_0(\frac{ab}{\sigma^2})da. \tag{5.4}$$

The definite integral in (5.4) can be solved analytically [29], so that $f_0^I(x)$ may be written as

$$f_0^I(x) = \frac{x(\gamma^2+1)}{\Gamma_I^2\gamma^2+\gamma^2+1}\exp\left\{-\frac{x^2(\gamma^2+1)+2\Gamma_I^2}{2(\Gamma_I^2\gamma^2+\gamma^2+1)}\right\}I_0\left\{\frac{x\Gamma_I\sqrt{2(\gamma^2+1)}}{\Gamma_I^2\gamma^2+\gamma^2+1}\right\}. \tag{5.5}$$

The conditional density for $R_{0,l}$ given that interference is absent, $f_0^N(x)$, is found by replacing Γ_I by Γ_N in (5.5). The quantities p_c, p_e, and $f_0^I(x)$ presented in (5.2), (5.3), and (5.5), are substituted into (4.9), (4.10), and (4.11) to find the conditional densities for Z given that interference is present. The quantities p_r, p_w, and $f_0^N(x)$ are substituted into (4.9), (4.10), and (4.11) to find the conditional densities for Z given that interference is absent. The resulting conditional densities are used to solve (4.6), except for the term $P(e\,;0,0)$. The variation on linear combining described in Section 4.3 is used for the situation in which all of the diversity receptions are rejected. The conditional error probability $P(e\,;0,0)$ is computed for this situation.

We compute the conditional error probability given that all of the diversity receptions are rejected by the ratio threshold test. Recall that for the l-th diversity reception, τ_l is equal to $\frac{R_{1,l}}{R_{0,l}}$. The diversity reception with the maximum value of τ_l or τ_l^{-1} (i. e., $\max_l(\max(\tau_l,\tau_l^{-1}))$) is used to decide which bit is sent for this situation. The density and distribution functions for τ_l for a diversity reception with interference and fading is found by replacing v_I by av_I in the density and distribution functions of (4.14a) and (4.14b), and then averaging with respect to the density in (5.1). For a diversity reception with Rician fading and interference, the conditional density and distribution functions are

$$f_\tau^I(t) = 2t(\gamma^2+1)\left[\frac{\Gamma_I^2\gamma^2+\gamma^2+1}{[d(t)]^2} + \frac{\Gamma_I^2(\gamma^2+1)}{[d(t)]^3}\right]\exp\left\{-\frac{\Gamma_I^2 t^2}{d(t)}\right\}, \tag{5.6a}$$

and

$$F_\tau^I(t) = 1 - \frac{\gamma^2+1}{d(t)}\exp\left\{-\frac{\Gamma_I^2 t^2}{d(t)}\right\}, \tag{5.6b}$$

where $d(t)=\Gamma_I^2\gamma^2t^2+(1+t^2)(\gamma^2+1)$. The conditional density and distribution functions given that interference is absent, $f_\tau^N(t)$ and $F_\tau^N(t)$, are found by substituting Γ_I by Γ_N in (5.6). The functions in (5.6), as well as $f_\tau^N(t)$ and $F_\tau^N(t)$, are substituted into (4.16) to find the probability of error given that all of the diversity receptions are rejected.

The probability of error for the ratio threshold test with the variation on majority logic decoding is found by using the derivations provided in Sections 4.2 and 4.3, and the quantities derived in the preceding paragraphs of this section. That is, the expressions in (5.2)-(5.3), (5.6), and the quantities corresponding to diversity receptions with interference absent are substituted in the appropriate places in the derivation presented in Sections 4.2 and 4.3.

5.4 Numerical Results

Figures 5.1 and 5.2 show the sensitivity of the probability of error to the threshold θ for the diversity combining schemes discussed in Section 5.3. The curves in Figure 5.1 correspond to the ratio threshold test with the variation on linear combining. Recall that a value of $\theta=4$ works well for the ratio threshold test with linear combining in the presence of partial-band interference ($\gamma^2=0$). The curves of Figure 5.1 illustrate that for $\rho=0.1$, a large value of θ (e. g., $\theta=6$) works well for a channel with both partial-band interference and fading. However, the error probability is not sensitive to θ.

The curves in Figure 5.2 correspond to the ratio threshold test with the variation on majority logic decoding. Note that for Rayleigh fading, a value of θ near 6 gives the minimum value of p_b in the curves shown. For Rician fading, smaller values of θ are better for small ρ. However, the error probability is not very sensitive to θ. We use $\theta=6$ for both linear combining and majority logic decoding in the examples that follow.

Figures 5.3 through 5.5 illustrate the performance of the ratio threshold test with the variation on linear combining for partial-band interference and fading. Note that the curve for $L=1$ is not plotted in Figure 5.3. This is because for $\beta_N^2=18.0$dB, a bit error probability of $p_b=0.01$ cannot be achieved no matter how large β_I^2 is. However, $p_b=0.01$ is achievable for diversity levels 2 and 3. An average received signal to quiescent noise ratio of 18dB is actually lower than the average received signal to interference ratio required in the example of Figure 5.3 for most values of ρ. Thus, the assumption that the quiescent noise level is much less than the interference level is violated in this example. For the examples in Figures 5.4 and 5.5, we allow a larger average received signal to quiescent noise ratio.

Recall that for the system with partial-band interference and no fading, diversity level 3 does not show improvement over diversity level 2 for the examples discussed in Chapter 4. For the ratio threshold test with the variation on linear combining with partial-band interference

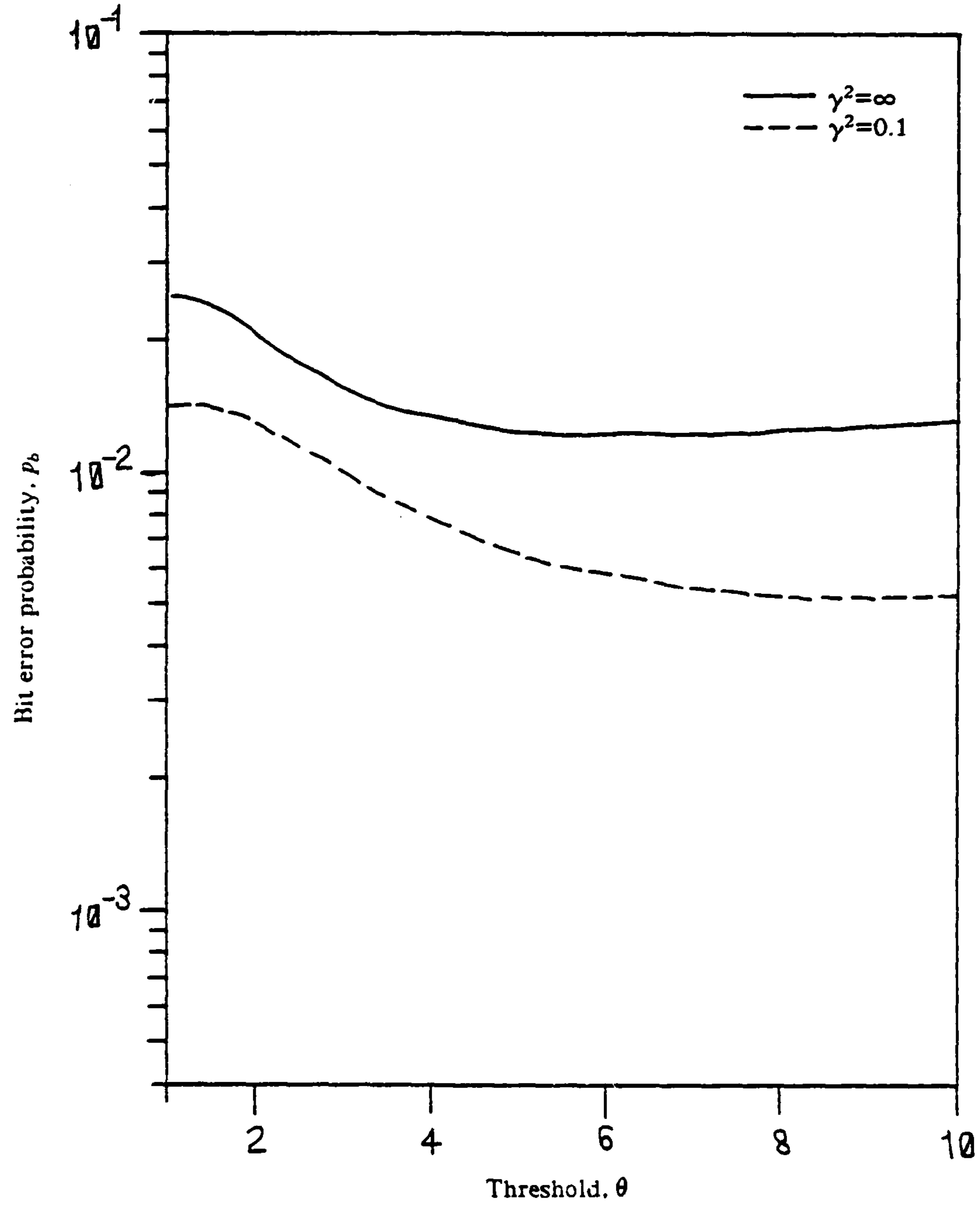

Figure 5.1. Probability of error p_b versus the threshold θ for $\rho=0.1$, $L=3$, $\beta_S^2=18.0$dB, and $\beta_I^2=12.0$dB for the ratio threshold test with the variation on linear combining

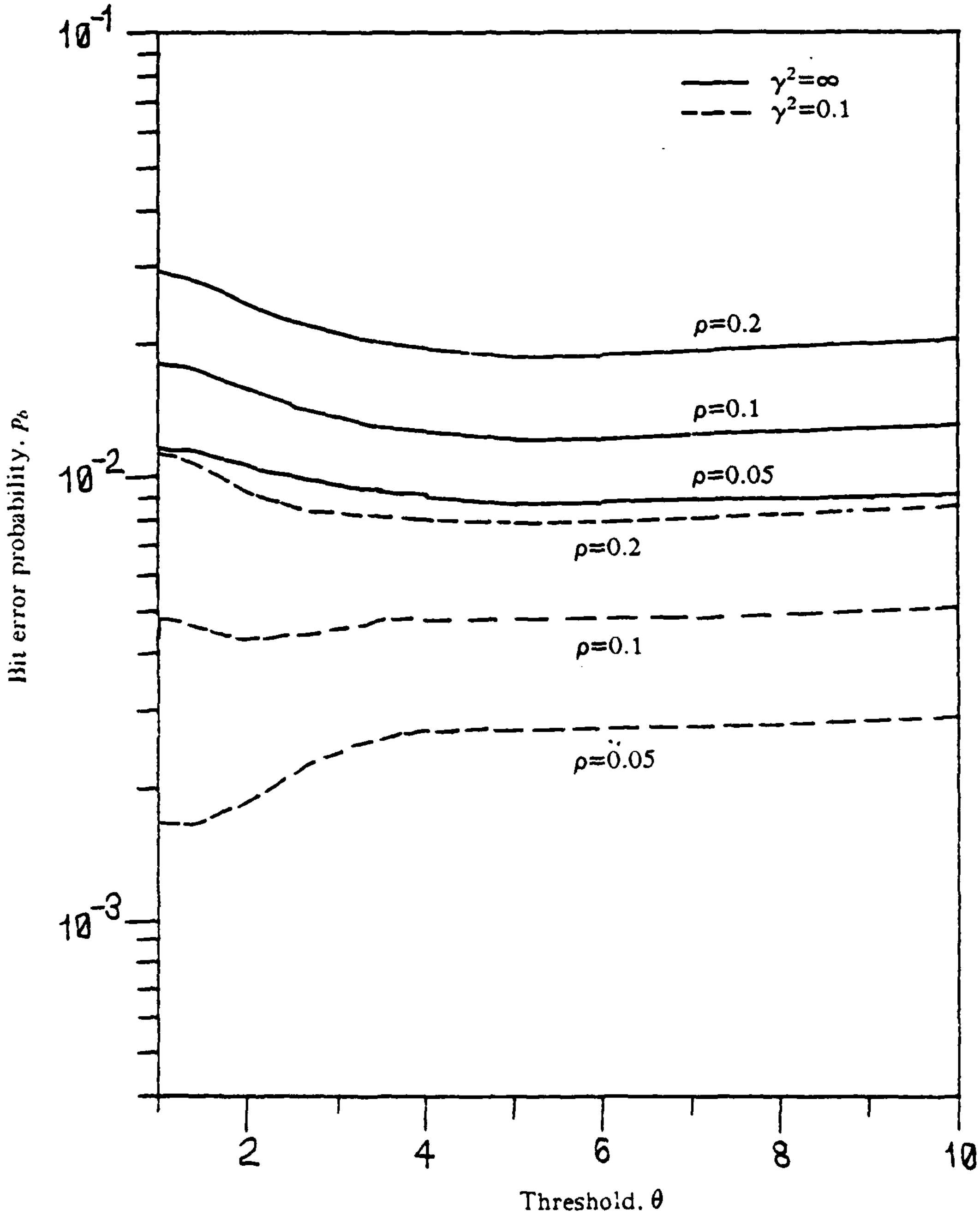

Figure 5.2. Probability of error p_b versus the threshold θ for small interference duty factors. $L=3$, β_{γ}^2=18.0dB, and β_I^2=12.0dB for the ratio threshold test with the variation on majority logic decoding

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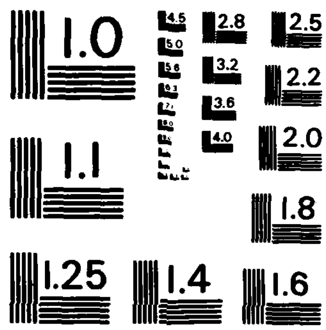

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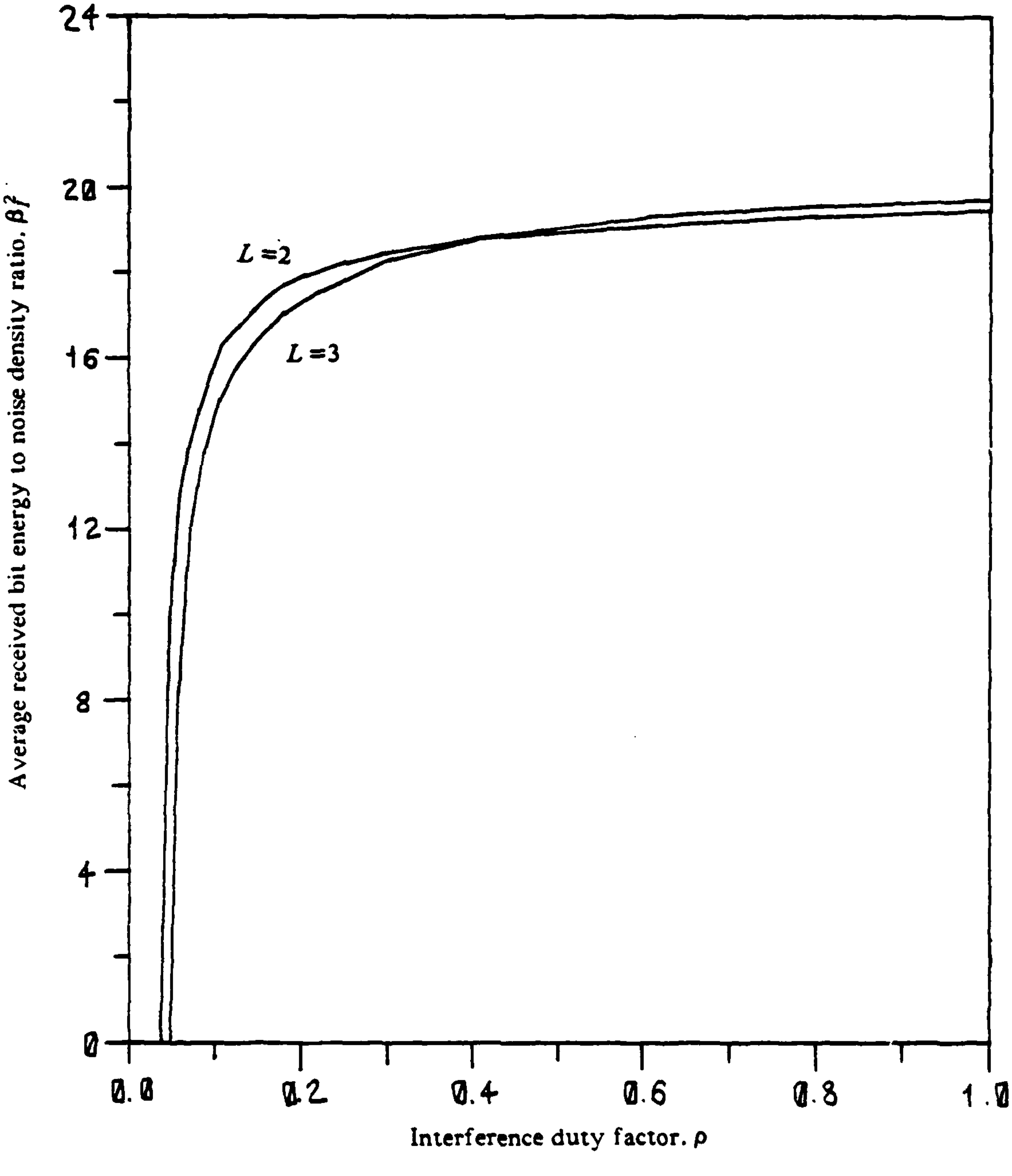

Figure 5.3. Average received bit energy to noise density ratio versus interference duty factor for p_b =0.01, β_N^2 =18.0dB, $\gamma^2=\infty$, and diversity levels 2 and 3 for the ratio threshold test with the variation on linear combining with θ=6

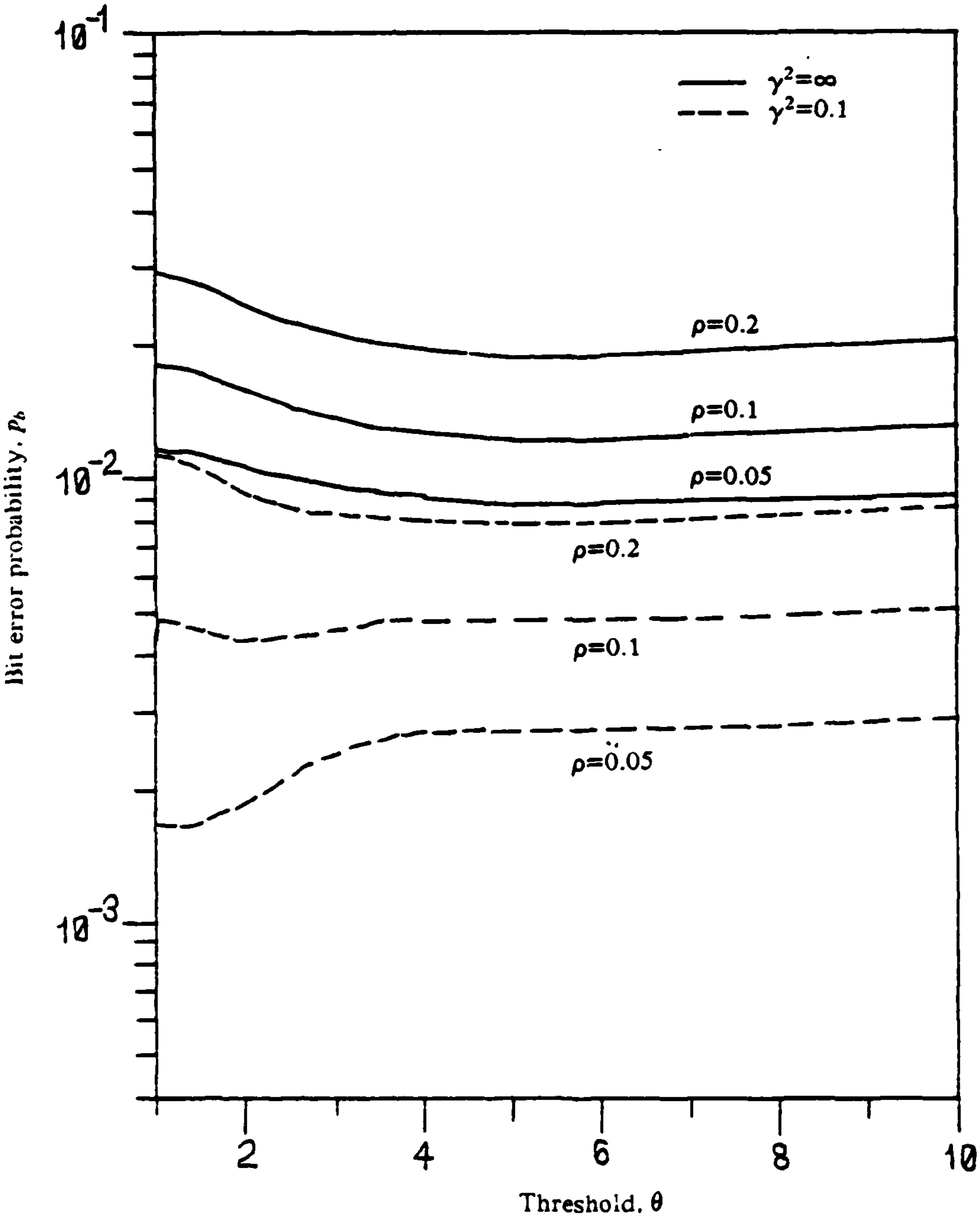

Figure 5.2. Probability of error p_b versus the threshold θ for small interference duty factors. $L=3$, $\beta_{s}^{2}=18.0$dB, and $\beta_{I}^{2}=12.0$dB for the ratio threshold test with the variation on majority logic decoding

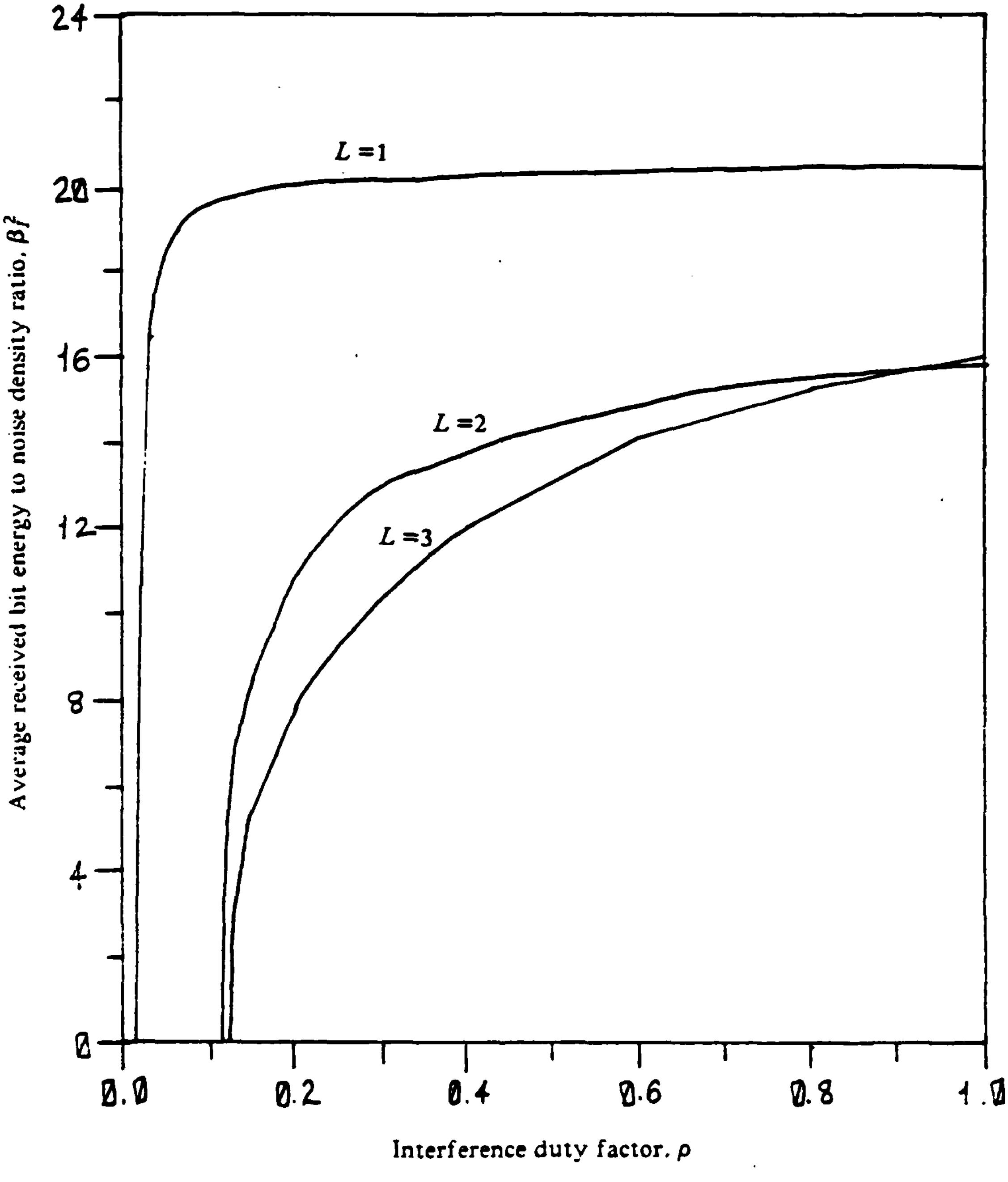

Figure 5.4. Average received bit energy to noise density ratio versus interference duty factor for p_b=0.01, β_N^2=30.0dB, γ^2=∞, and diversity levels 1 through 3 for the ratio threshold test with the variation on linear combining with θ=6

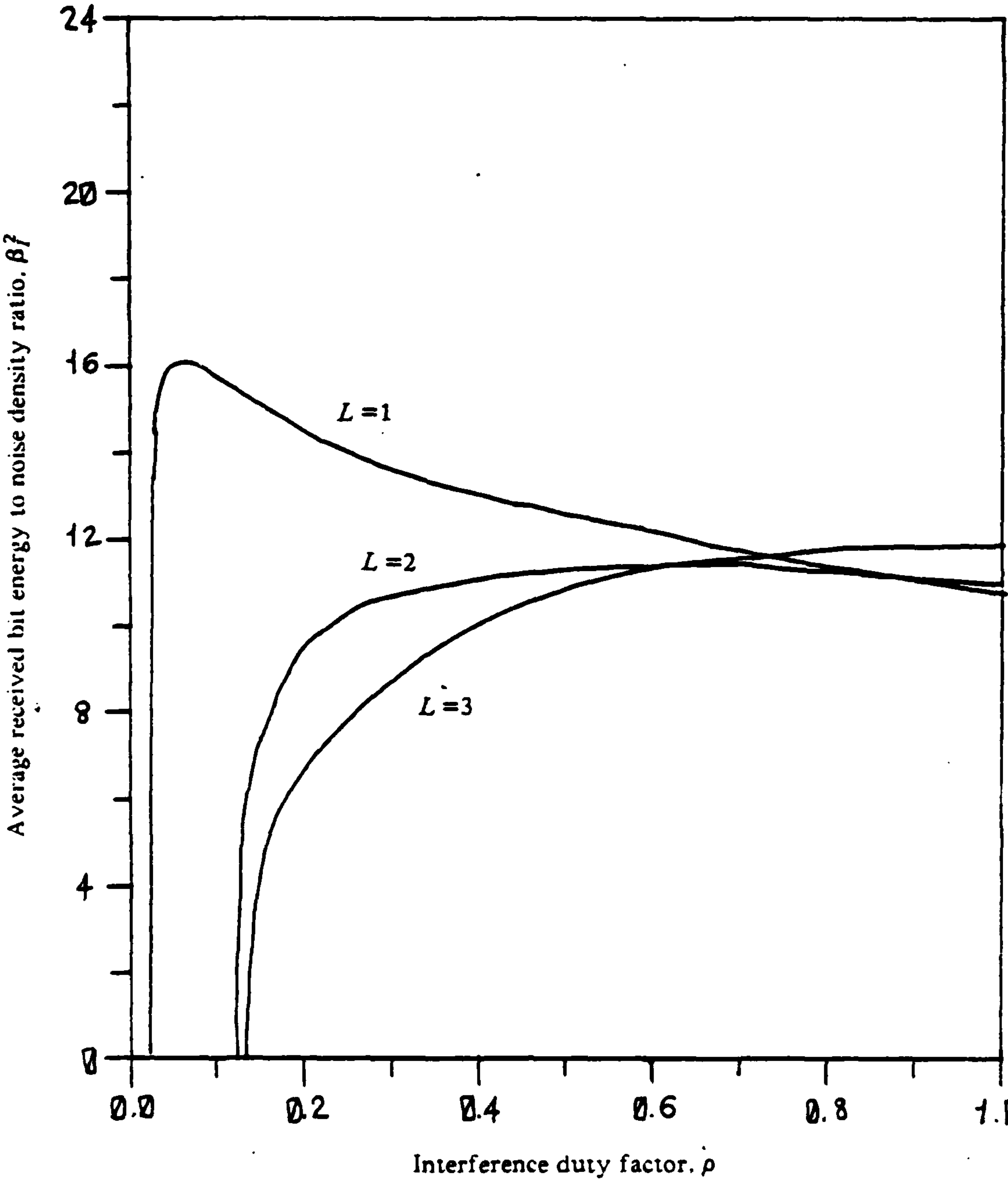

Figure 5.5. Average received bit energy to noise density ratio versus interference duty factor for $p_b=0.01$, $\beta_N^2=30.0$dB, $\gamma^2=0.1$, and diversity levels 1 through 3 for the ratio threshold test with the variation on linear combining with $\theta=6$

and fading, increasing the diversity level above $L=2$ leads to improvement in ρ_{min}. However, increasing the diversity level also causes the signal to noise ratio requirement for large interference duty factors to increase. This increase is less than 1.5dB in the examples shown.

Figures 5.6 through 5.8 give the performance of the ratio threshold test with the variation on majority logic decoding for partial-band interference and fading. In Figure 5.6, ρ_{min} is smaller for diversity level 4 than it is for diversity level 3. The performance is poor for all diversity levels because the signal to quiescent noise ratio is too small. Large improvement in ρ_{min} is seen in Figures 5.7 and 5.8, as the diversity level increases from 1 to 4. The increase in the signal to noise ratio requirement for large interference duty factors is less than 1.5dB for each increment in the diversity level.

Note that the curves for diversity level 1 are the same in Figures 5.6 through 5.8 as they are in the corresponding examples in Figures 5.3 through 5.5. That is, for $L=1$, the schemes that use the variation are all the same and, in fact, are independent of θ. The decision is always based on one diversity reception whether that diversity reception is accepted or rejected by the ratio threshold test.

Also, note that for Rayleigh fading ($\gamma^2=\infty$), the maximum signal to noise ratio required over the range of interference duty factors occurs at $\rho=1$. This is demonstrated in Figures 5.3, 5.4, 5.6, and 5.7. That is, full band interference is the worst-case partial-band interference. This fact is also observed in [10] for the ratio threshold test with majority logic decoding (no variation), and is demonstrated in other work, such as [30].

Figure 5.9 presents a comparison between the variation on linear combining and the variation on majority logic decoding for Rayleigh fading. Figure 5.9 also includes the performance of square-law combining for a receiver with perfect side information (shown as the dotted curve), which, in Rayleigh fading, is the optimum combining technique. Majority logic decoding is closer to square-law combining at small ρ, and linear combining is closer to square-law combining at large ρ. Majority logic decoding has a significantly better value of ρ_{min} than linear

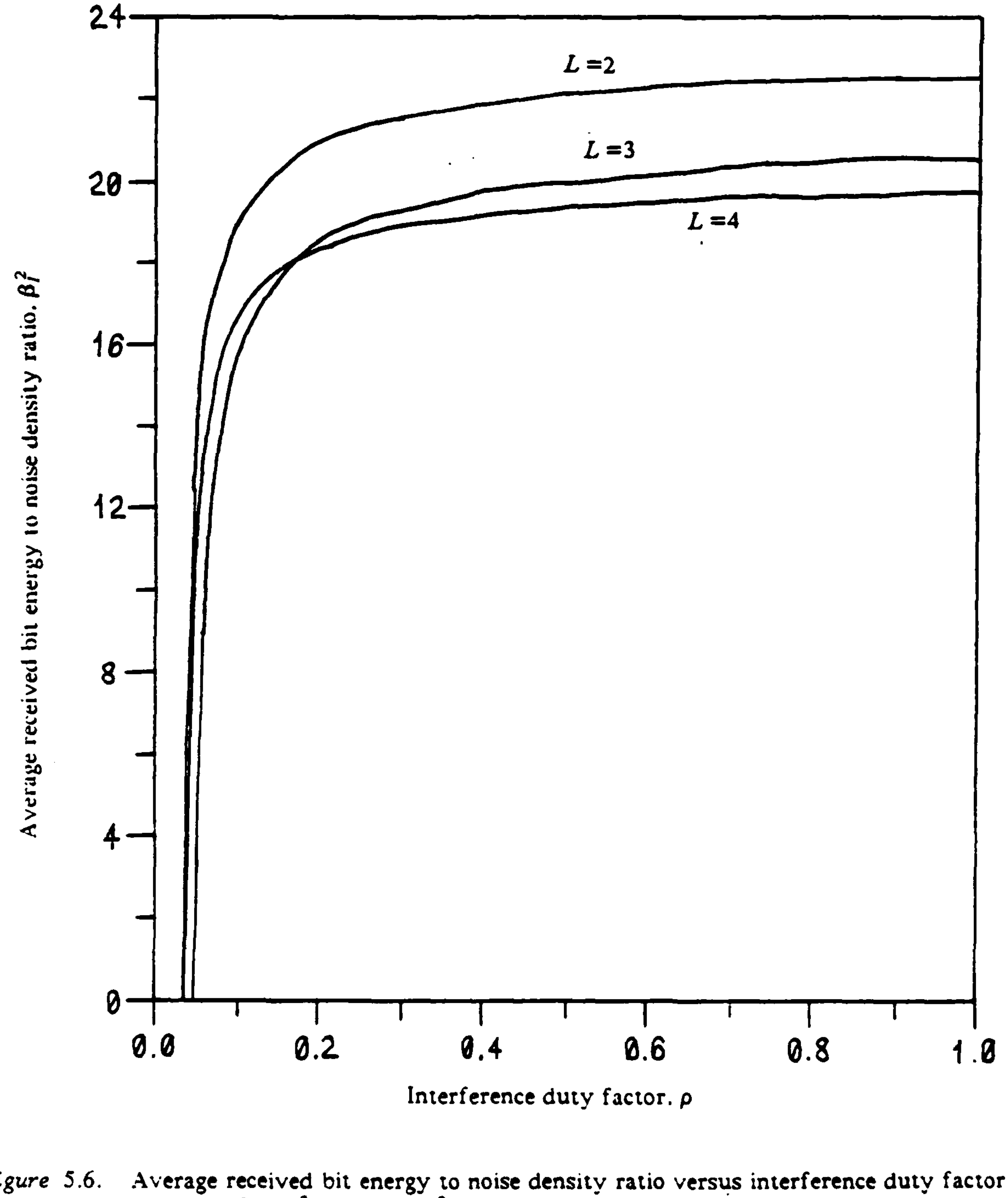

Figure 5.6. Average received bit energy to noise density ratio versus interference duty factor for p_b =0.01, β_N^2=18.0dB, $\gamma^2=\infty$, and diversity levels 2 through 4 for the ratio threshold test with the variation on majority logic decoding with θ=6

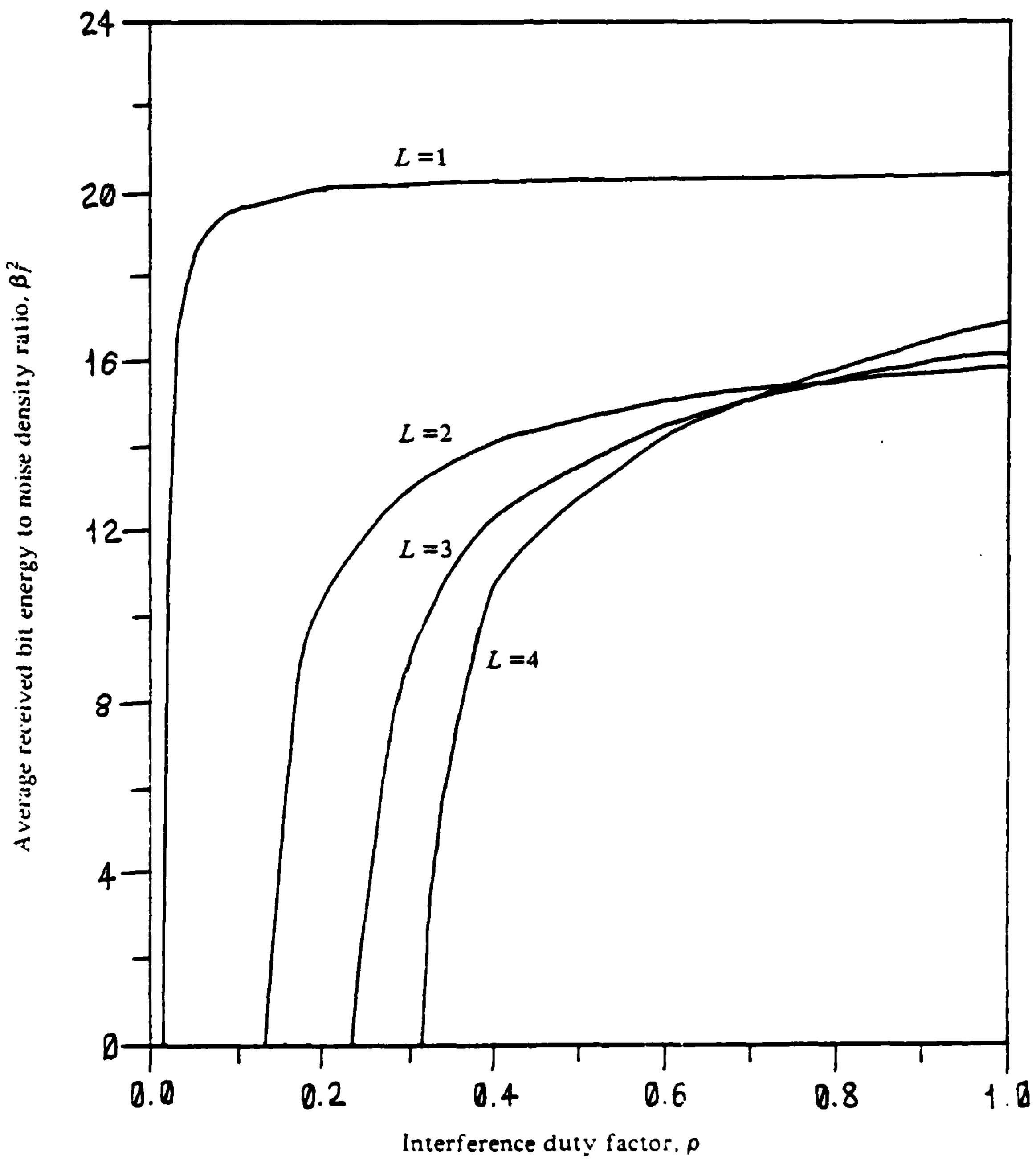

Figure 5.7. Average received bit energy to noise density ratio versus interference duty factor for p_b =0.01, β_N^2=30.0dB, $\gamma^2=\infty$, and diversity levels 1 through 4 for the ratio threshold test with the variation on majority logic decoding with θ=6

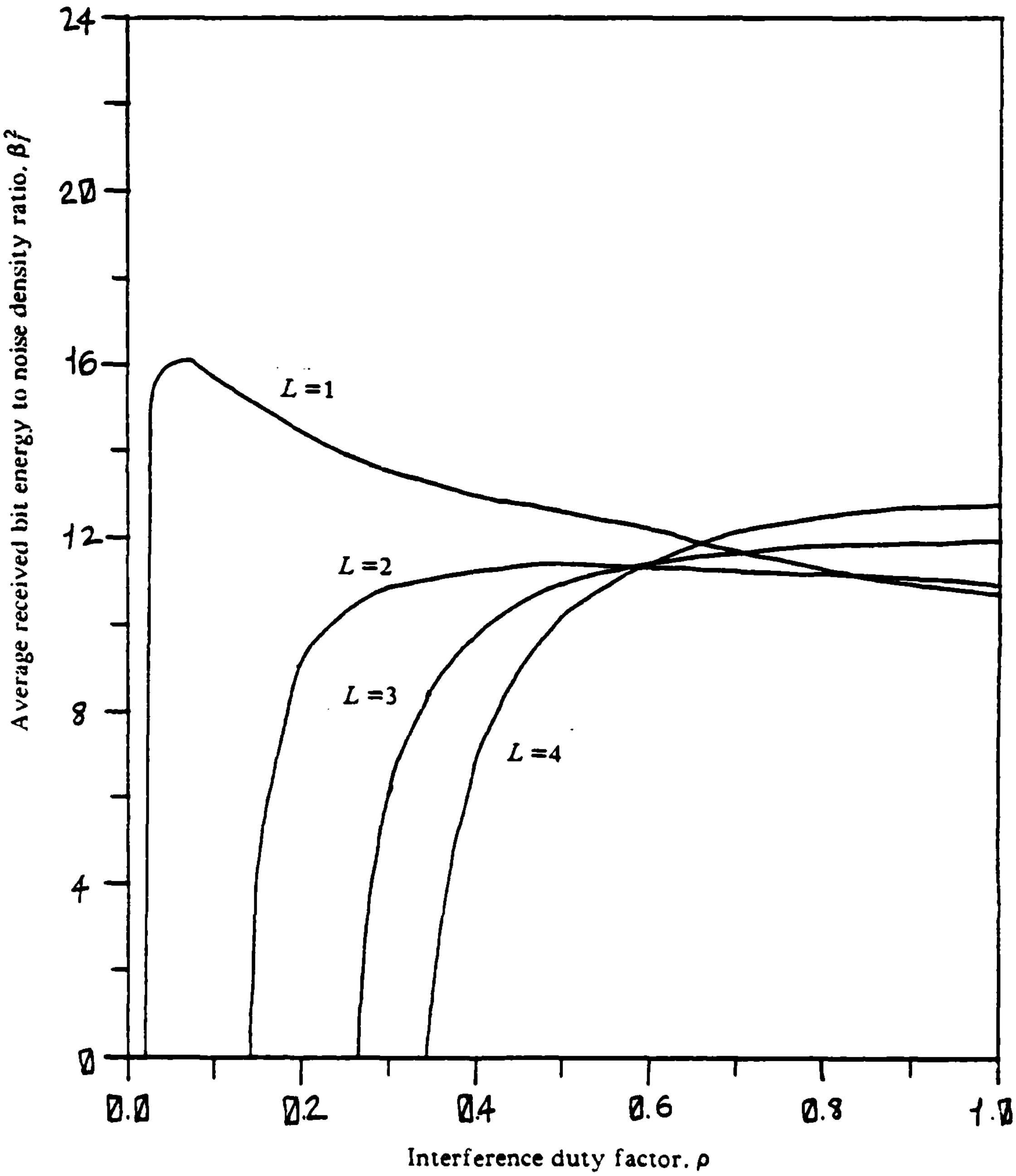

Figure 5.8. Average received bit energy to noise density ratio versus interference duty factor for $p_b=0.01$, $\beta_N^2=30.0$dB, $\gamma^2=0.1$, and diversity levels 1 through 4 for the ratio threshold test with the variation on majority logic decoding with $\theta=6$

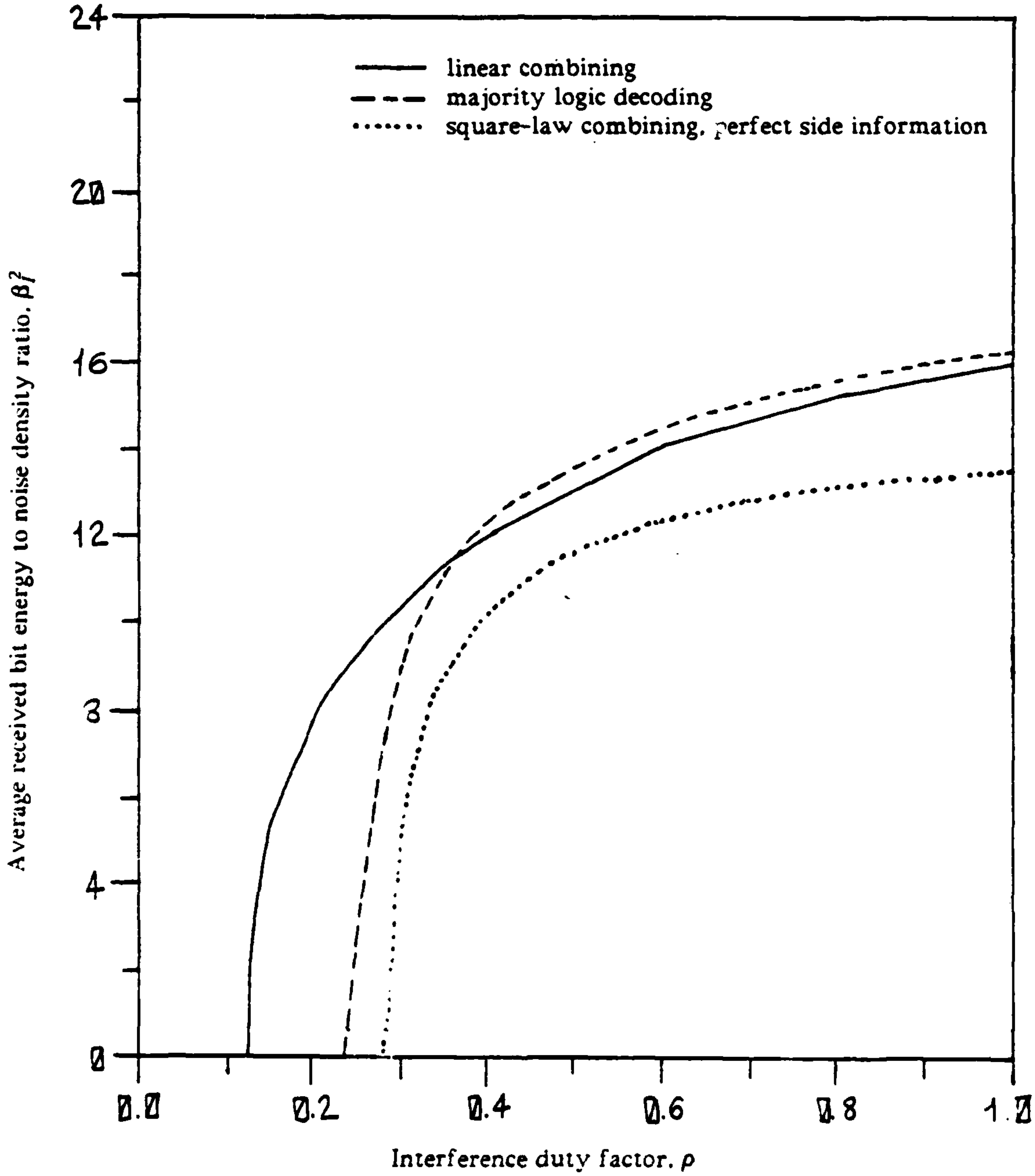

Figure 5.9. Average received bit energy to noise density ratio versus interference duty factor for $L=3$, $p_5=0.01$, $\beta_N^2=30.0$dB, $\gamma^2=\infty$, and $\theta=6$ for the ratio threshold test with the variation on linear combining versus the ratio threshold test with the variation on majority logic decoding versus square-law combining with perfect side information combining (more than 0.1 larger in this example), without much tradeoff in the signal to noise ratio requirement at large ρ. (The difference in the signal to noise ratio requirement is about 0.5dB in this example.) Thus, the ratio threshold test with the variation on majority logic decoding can be considered better, or more "robust," than the scheme with linear combining.

5.5 The Ratio Threshold Test for M-ary Orthogonal Signaling

The procedure for calculating the probability of error for the scheme with the ratio threshold test and M-ary orthogonal signaling was presented in Section 4.4. The probability of error for this scheme in the presence of partial-band interference and fading is found by using the same procedure. The only quantities that need to be averaged with respect to the density in (5.1) are the conditional density function for $R_{0,l}$ given that interference is present, namely, $f_0^I(x)$, and the conditional density function for $R_{0,l}$ given that interference is absent, namely, $f_0^N(x)$. But, the density $f_0^I(x)$ is already presented in (5.5), and $f_0^N(x)$ is found by replacing Γ_I by Γ_N in (5.5). These densities are substituted into the appropriate places in Section 4.4 to calculate the probability of error for the ratio threshold test with M-ary orthogonal signaling.

Figure 5.10 presents a comparison of the performance of the ratio threshold test and that of the optimum combining technique for 32-ary orthogonal signaling and Rayleigh fading. The optimum technique is square-law combining for a receiver with perfect side information. The value of ρ_{min} for the ratio threshold test is about 0.5 less than that for the square-law combining. Thus, the narrowband interference rejection of the ratio threshold test is good. As much as 55 percent of the frequency band must have interference for the symbol error probability to be larger than 0.1 for diversity level 5. For large ρ, the signal to noise ratio requirement for the ratio threshold test is about 2.5dB more than it is for square-law combining. However, note that these numerical results are for the scheme that makes a random decision for the situation in which all the diversity receptions are rejected. If the variation on this scheme is employed, the signal to noise ratio requirement at large ρ will be reduced.

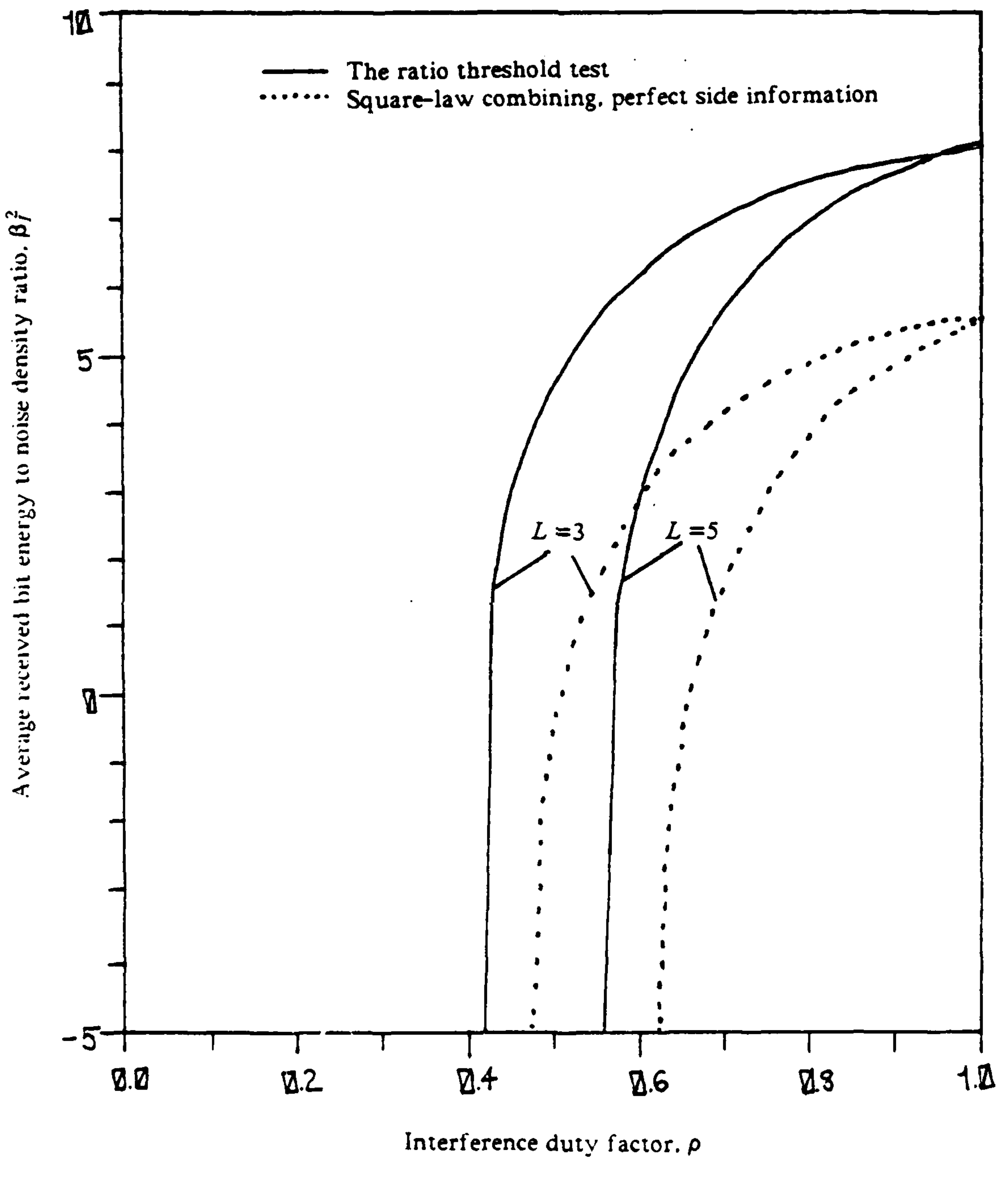

Figure 5.10. Average received bit energy to noise density ratio versus interference duty factor for $p_s=0.1$, $\beta_N^2=30.0$dB, $\gamma^2=\infty$, and $\theta=1.3$ for the ratio threshold test for 32-ary orthogonal signaling versus square-law combining with perfect side information

Figure 5.11 shows the performance of the ratio threshold test for a system with 32-ary orthogonal signaling and diversity levels 3, 5, and 7, for Rician fading with $\gamma^2=0.1$. The value of ρ_{min} increases with the diversity level, and it is 0.65 for $L=7$. The curves in this figure demonstrate the good narrowband interference rejection capability of FH communication systems that employ diversity transmission and the ratio threshold test.

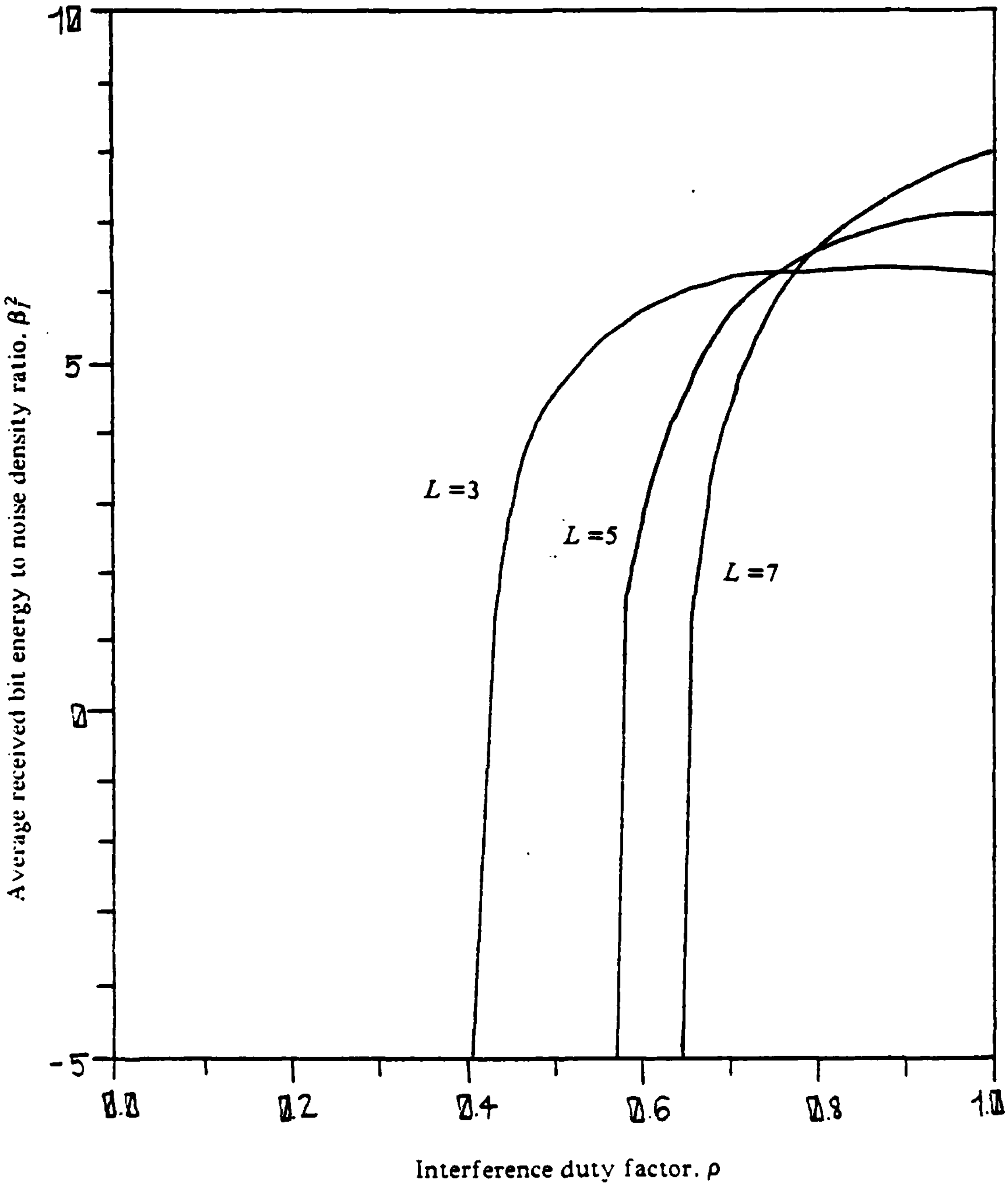

Figure 5.11. Average received bit energy to noise density ratio versus interference duty factor for $p_s = 0.1$, $\beta_N^2 = 30.0$dB, $\gamma^2 = 0.1$, and $\theta = 1.3$ for the ratio threshold test for 32-ary orthogonal signaling

CHAPTER 6

CONCLUSIONS

In this thesis, we have analyzed several diversity combining schemes for frequency-hop communications in the presence of partial-band interference and fading. Each diversity combining technique has been evaluated on its narrowband interference rejection capability and on its signal to noise ratio requirement over all values of interference duty factors.

We have calculated the exact probability of error for the optimum diversity combining scheme for a receiver in which perfect side information is available. Several suboptimum diversity combining schemes for receivers with perfect side information have been compared to the optimum combining technique, and it has been found that all of these schemes perform nearly the same. It is the availability of the perfect side information that allows all of these schemes to perform well in the presence of partial-band interference.

Several diversity combining schemes have been analyzed for receivers without side information. Clipped linear combining was shown to be effective against narrowband interference. It can perform nearly as well as a receiver with perfect side information. A sensitivity analysis of clipped linear combining has demonstrated that its performance may be unpredictable if the there are large deviations in the signal output voltage (e. g., more than 3dB).

The ratio threshold test, used in conjunction with diversity combining, has been shown to be another effective technique for partial-band interference. The ratio threshold test with majority logic decoding and with M-ary orthogonal signaling work well in terms of narrowband interference rejection. However, in some of the examples shown, the signal to noise ratio requirement is significantly higher for these schemes than it is for the optimum combining scheme. Although the ratio threshold test with linear combining provides some narrowband interference rejection, it does not do so as effectively as the other combining techniques considered.

The diversity combining schemes that employ the ratio threshold test have been analyzed for partial-band interference and nonselective Rician fading. It has been shown that the ratio threshold test with M-ary orthogonal signaling (including $M=2$ and majority logic decoding) provides good narrowband interference rejection. The signal to noise ratio requirement of this scheme for large values of the interference duty factor is much higher (more than 3 dB) than the requirement for square-law combining with perfect side information, the optimum combining technique for Rayleigh fading. Although the ratio threshold test with diversity combining does not achieve the optimum performance, it is an effective, as well as practical, scheme for use in FH systems subject to partial-band interference and fading.

There are many variations on the diversity combining techniques that use ratio statistics. One technique that has not been discussed in this thesis is *ratio statistic combining*. In this scheme, a ratio statistic is formed for each symbol on each diversity reception. The ratio statistics are added, and the symbol with the largest sum is chosen. The analysis of this scheme and of other variations on the ratio threshold test is a topic for further research.

Although some results have been given in this thesis for general (non-Gaussian) interference, as well as for Gaussian partial-band interference, there are other models for the interference that could be explored in future work. For example, models for multiple-access interference could be studied as a class of partial-band interference. Also, the partial-band interference could be modeled by a generalized Gaussian process [31]. Finally, for the analysis of communications via fading channels, the partial-band interference could be modeled as a fading process.

APPENDIX

METHODS USED TO VERIFY NUMERICAL RESULTS

Most of the numerical results presented in this thesis required extensive numerical computation. The purpose of this Appendix is to describe the tools we used in obtaining the data and how the programs used to compute the data were verified.

The computer programs were written in Fortran. The computations were done on Digital Equipment Corporation VAX 11/780 computers. In addition, a Floating Point System's Array Processor (AP) with its scientific subroutines was employed. The subroutines on the AP that were used include vector convolution, vector Simpson's integral, vector multiplication, and vector multiplication by a scalar. The AP was very instrumental in saving computation time. In fact, in many cases, there was a savings of computation time by a factor of 10 for programs that used the AP when compared to programs that did not use the AP.

Examples of curves that required the most computation time include the curves for the ratio threshold test with the variation on linear combining in Figures 4.5, 5.3, and 5.5 for $L=3$. The results in Table 2.2 corresponding to the optimum diversity combining technique for $L=5$, also used extensive computation time. Other numerical results used less computation time.

To verify that the computer programs were correct, tests were run for special cases of the problem for which results are found in the literature. In addition, special cases that could also be compared with other programs that were written independently, were also tested for agreement between the programs. To explain in more detail how the numerical results were verified, we use as an example the program written for the ratio threshold test with majority logic in the presence of Rician fading. The case with $L=1$, $\theta=1$, and $\rho=1$, was checked against curves given in [23] for various γ^2, and β_I^2. The case with $\theta=1$, $\rho=1$ and $\gamma^2=\infty$ (or $1/\gamma^2=0$) was checked against the curves in Figure 5.5 in [10] for $L=2$, 4, and 6. The program was tested against the program for the ratio threshold test with majority logic decoding, no fading, for various values of ρ, θ, L, E_b/N_I, and E_b/N_0, by letting γ^2 become small (e.g., $1/\gamma^2=1{,}000{,}000$).

As another example of how the numerical results were verified, consider the program written for clipped linear combining. The results obtained for large clipping levels (e.g., $C=7\beta$) were compared to the results obtained from the program that computes the performance of the ratio threshold test with linear combining with $\theta=1$ for binary orthogonal signaling. Also, the results for large clipping levels were compared with the results from the program for standard linear combining with no side information for M-ary orthogonal signaling. These tests were performed for various diversity levels. The program was also checked against the analytical example discussed in Section 3.3, for $M=2$ and $L=2$.

Similar methods were used to test all other programs that generate data for this thesis. Although many of the numerical results presented for the values of the signal to noise ratio are accurate to one hundredth of a dB, in general we claim that our results are accurate to one tenth of a dB.

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VITA

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"Frequency Spacing in FSK Frequency-Hopped Spread-Spectrum Communications over Selective Fading Channels," *Proc. 17th. Conf. Information Sciences and Systems*, The Johns Hopkins University, Baltimore, MD, pp. 749-754, March 1983. (with M. B. Pursley)

"Diversity Combining for Frequency-Hop Spread-Spectrum Communications with Partial-Band Interference," *Proc. of the 1984 IEEE Military Communications Conference*, Los Angeles, CA, pp. 464-467, October 1984. (with M. B. Pursley)

"Diversity Combining and Viterbi's Ratio Threshold Technique for Frequency-Hop Communications in the Presence of Partial-Band Interference," *Proc. 19th. Conf. Information Sciences and Systems*, The Johns Hopkins University, Baltimore, MD, pp. 532-537, March 1985. (with M. B. Pursley)

"A Comparison of Diversity Combining Techniques for Frequency-Hop Spread-Spectrum Communications with Partial-Band Interference," to be presented at the *1985 IEEE Military Communications Conference*, Boston, MA, October 1985. (with M. B. Pursley)

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